CHAPTER 33 Alternating Currents

Answers to Understanding the Concepts Questions

1. The core material plays a major role in linking the two coils, so it should efficiently amplify the magnetic field as well as confine it to the linking region. This means that the energies within these materials are substantial, and there are forces present that require that the core have a sufficient mechanical strength. Cores used in demonstrations for introductory physics often involve thick wires tied into a spiral for mechanical strength.

2. If you go to a foreign country where the voltage of the standard power source is higher than 110 V (e. g. Europe and most of Asia), and you need to take along with you an electronic device that operates on 110 V, then you need a step-down transformer than converts 220 V to 110 V. Similarly, if a foreign visitor needs to do the opposite by bringing in a device that operates on 220 V to be used in this country, a step-down transformer would be necessary. As a matter of fact, nearly all the electronic gadgets you use, from laptop computers to cell phone charger to portable CD players, etc., operate on a voltage other than 110 V (usually much lower), so they feature a built-in transformer that comes along with the power supply. Also, a neighborhood step-down transformer the size of a small table is used to lower the voltage from power lines from hundreds of thousands of volts to several hundred volts.

3. At very high frequencies the charging/discharging processes reverse back and forth very rapidly, so the maximum charge accumulated on the capacitor is very small, and therefore the maximum voltage difference across it is almost zero. Indeed, the reactance of a capacitor of capacitance C is $1/\omega C$, which approaches zero as ω becomes very high.

4. A superconductor is superconductive only when the current density stays below a certain critical value. If it is driven by an oscillatory power source at resonance frequency, eventually the current density in the superconductor would exceed this critical value and the material reverts to a normal conductor, with a finite amount of resistance. So the current can never actually reach infinity.

6. The lightbulb is essentially a resistor. So the circuit in question is an *RLC* circuit, which has a resonance angular frequency $\omega_0 = 1/(LC)^{1/2}$. The rms current is the greatest at this frequency, and hence the lightbulb glows the brightest when $\omega = \omega_0$.

7. The current alternates, sometimes negative and sometimes positive. In the harmonic form, every time it has a certain positive value there is another time when the current has the same value, only negative. The average of the current itself is simply zero, and this does not tell us much about important features we want to study.

8. The loop rule equation that gives the relationship between the emf of the power supply and the corresponding current in the circuit is a linear equation, so it satisfies the linear superposition principle. This means that, if the emf that powers the circuit consists of two separate terms, each alone generating a current I_1 and I_2, respectively, then the total current in the circuit is $I_1 + I_2$.

9. In principle, a transformer can indeed be reversed, whereupon a step-up transformer becomes a step-down transformer, and vice versa. One must observe the power rating so as not to overload the circuit, however.

10. The lamp is a resistor, and the "brightness" with which it burns is a measure of the energy loss within it, hence of the energy loss in a driven RC circuit. Recall also that a capacitor acts like an open switch for direct current, but passes alternating current more and more easily as the frequency increases. With these statements we can see that (a) is false except when the generator frequency is zero (direct current). We would also reckon that (b), but not (c), is true. We can verify this by recalling Eq. (33-38), which shows that for a driven RC circuit with an emf amplitude V_0, the power loss is given by $V_0^2R/2[(1/\omega C)^2+R^2]$. As ω increases, this loss increases.

11. A capacitor acts like an open circuit at very low frequencies and like a wire at high frequencies. So at very low frequencies the capacitor can be effectively removed from the circuit, as practically all the current would flow through the lightbulb, which burns brightly. At high frequencies the voltage across the capacitor, which is the same as that across the lightbulb, is almost zero. The lightbulb would not burn, or be very dim, at high frequencies. So the correct answer is (b).

12. The maximum current is most often determined by capacity of the appliance to dissipate thermal energy, and that in turn is a function of the internal resistance. The result of too much current through a resistor is so much thermal energy generated that the device melts! Other factors that may enter have to do with the stability of semiconductor materials and of microcircuits to withstand the changes generated by elevated temperatures.

13. An inductor is suitable for the job. As the power is turned on the current starts from zero. The inductor prevents the current from changing too fast (from zero), so that limits current spikes. Also, the voltage across the inductor is $L(\mathrm{d}I/\mathrm{d}t)$, which can exceed that of the power supply just after the power is turned on, as $\mathrm{d}I/\mathrm{d}t$ is significant over the short time duration before the current reaches a steady value. This provides the necessary large voltage that starts up the light.

14. The power delivered by the heater is $P=V^2/R$. A higher voltage V results in a higher power.

15. Resistors in series are equivalent to a resistance that is the sum of the individual resistances, so that the formula in question is indeed correct in this case. And when individual capacitors are placed in series, their inverses add to an equivalent inverse capacitor, so that once again the impedances add. A similar result holds for inductance. We can see all this from Eq. (33-48). The reason the individual impedances do not add when more than one element is involved has to with the fact that the different terms in effect enter with different phases. The phase does not matter when only one type of element is involved, but the problem is made more difficult when the phase is present. In fact, when the impedance is written using complex numbers, there is a very useful method to handle phase complications: impedances connected in series are indeed equivalent to a single impedance whose (complex) value is the sum of the (complex) individual impedances. In this language Eq. (33-48) is an expression for the magnitude of the impedance.

16. In a DC circuit $\omega=0$, so $X_C=1/\omega C$ becomes infinity. Physically, what this means is that, since the current never reverses its direction of flow in a DC circuit, the capacitor will sooner or later be fully charged. Once the charging process is over, no current can flow in the circuit. This is reflected in the equation $I_C=V_C/X_C=0$ as X_C becomes infinity.

17. Just because a capacitor's impedance for low frequency is large does not mean it is infinite, and just because an inductor's impedance for high frequency is large does not mean it is infinite either. There is plenty of room for current to pass for frequencies that are neither zero nor infinitely large.

18. Since $X_L=\omega L$, as ω doubles so does L. The answer is (c).

19. Using antenna wires of the same impedance Z ensures the delivery of maximum power — see the discussion on impedance matching in the textbook.

20. There are two features of appliance plugs worth noting. Each are associated with safety. First, some appliances have three-prong plugs. The third prong is a grounding line. It connects to ground in a wall receptacle which connects that prong to a metal pipe or a grounding cable somewhere. Its purpose is to make sure that a loose wire or extraneous piece of metal in the appliance will not make the outer case "hot," but rather connects the hot wires to ground and thereby causes a fuse to blow. The second feature is something that nearly all new appliances with two-prong plugs have, and that is that one of the prongs takes a flat spade form that can only enter the wall receptacle in one sense. The spade-accepting receptacle is connected to ground, while the second receptacle is "hot," with a potential that varies from positive to negative. Thus the spade-shaped prong plays much the same role as the third prong in the first case we discussed. Unfortunately, older buildings may not be equipped to take these plugs, and in these cases many people will file the spade shape down. User beware!

21. When the frequency of the emf is equal to the resonance frequency of the *LRC* loop, i.e., $f_0 = 1/[2\pi(LC)^{1/2}]$, the amplitude of the current in the loop will be the greatest, so bulbs 1 and 2 will be bright, while bulb 3 will be dim since the net current going through it, $I = I_L + I_C$, is close to zero at that frequency.

22. Yes. All CRT's have a high-voltage source that accelerates the electron beams, so a step-up transformer is necessary to increase the voltage from 110 V to a considerably higher value.

23. It is always possible to use transformers to change the voltage amplitude when it becomes desirable. The possible amplitude of a generator has to do with factors such as the speed of rotating turbine blades and the strength of the magnetic field within which the rotors turn. Most often this amplitude is considerably less than the high voltages that make for efficient transport of electrical energy, and transformers are used at the transition area between generating plant and power lines.

24. No, the current is not carried by sparks. As the voltage across the capacitor plates change, so must the charges carried by each plate; as $Q = CV$. As a result charges flow in and out of the plates, and hence the current.

Solutions to Problems

1. We find the voltage in the secondary from

   $\mathcal{E}_2/\mathcal{E}_1 = N_2/N_1;$

   $\mathcal{E}_2/(600\text{ V}) = (1500\text{ turns})/(100\text{ turns})$, which gives

   $\mathcal{E}_2 = \boxed{9000\text{ V}}.$

2. For the power delivered by the transmission line, we have $P = IV$. The power lost to Joule heating is

   $P_{\text{lost}} = I^2R = (P/V)^2R.$

   The cost of this lost power is

   $\text{cost} = (\text{rate})P_{\text{lost}} = (\text{rate})(P/V)^2R.$

   (a) For a voltage of 750,000 V, we have

   $\text{cost} = (\$0.18/\text{kWh})[5.0\times10^5\text{ W})/(7.5\times10^5\text{ V})]^2 \times (150\ \Omega)(10^{-3}\text{ kW/W})(1\text{ yr})(365\text{ days/yr})(24\text{ h/day}) = \boxed{\$105}.$

   (b) For a voltage of 1440 V, we have

   $\text{cost} = (\$0.18/\text{kWh})[(5.0\times10^6\text{ W})/(1440\text{ V})]^2(150\ \Omega)(10^{-3}\text{ kW/W})(1\text{ yr})(365\text{ days/yr})(24\text{ h/day}) = \boxed{\$2.9\times10^7}.$

$\boxed{3}$. We find the number of turns in the secondary from

   $\mathcal{E}_2/\mathcal{E}_1 = N_2/N_1;$

   $(12\text{V})/(110\text{ V}) = N_2/(550\text{ turns})$, which gives

   $N_2 = \boxed{60\text{ turns}}.$

4. We find the current in the primary coil from

   $\mathcal{E}_2/\mathcal{E}_1 = N_2/N_1 = I_1/I_2;$

   $(24\text{ V})/(115\text{ V}) = I_1/(2.0\text{ A})$, which gives

   $I_1 = \boxed{0.42\text{ A}}.$

5. When all turns of the secondary coil are used, the voltages of both coils are the same, so there must be 1200 turns on the primary coil. We find the number of turns of the secondary to be used for 45 V from

   $\mathcal{E}_2/\mathcal{E}_1 = N_2/N_1;$

   $(45\text{V})/(120\text{ V}) = N_2/(1200\text{ turns})$, which gives

   $N_2 = \boxed{450\text{ turns}}.$

   We find the current in the secondary coil from

   $\mathcal{E}_2/\mathcal{E}_1 = I_1/I_2;$

   $(45\text{V})/(120\text{ V}) = (10\text{ A})/I_2$, which gives

   $I_2 = \boxed{27\text{ A}}.$

6. We find the number of turns on the first coil from its self inductance:

   $L_1 = \mu_0 N_1^2 A/\ell;$

   $74\times10^{-3}\text{ H} = (4\pi\times10^{-7}\text{ T}\cdot\text{m/A})N_1^2(35\times10^{-4}\text{m}^2)/(0.20\text{ m})$, which gives

   $N_1 = \boxed{1.8\times10^3\text{ turns}}.$

   We find the number of turns on the second coil from the mutual inductance:

   $M = \mu_0 N_1 N_2 A/\ell = L_1 N_2/N_1;$

   $43.5\times10^{-3}\text{ H} = (74\times10^{-3}\text{ H})N_2/(1.8\times10^3)$, which gives

   $N_2 = \boxed{1.1\times10^3\text{ turns}}.$

7. We find the voltage of the secondary coil from

$\mathcal{E}_2/\mathcal{E}_1 = N_2/N_1$;

$\mathcal{E}_2/(220\text{ V}) = (40\text{ turns})/(1200\text{ turns})$, which gives $\mathcal{E}_2 = 7.33\text{ V}$.

We find the current in the secondary coil from

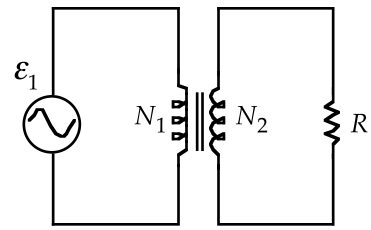
$P_2 = I_2{}^2R = I_2\mathcal{E}_2$;

$88\text{ W} = I_2(7.33\text{ V})$, which gives $I_2 = 12\text{ A}$.

We find the current in the primary coil from

$N_2/N_1 = I_1/I_2$;

$(40\text{ turns})/(1200\text{ turns}) = I_1/(12\text{ A})$, which gives $I_1 = \boxed{0.40\text{ A}}$.



8. (a) We find the voltage of the secondary from

$\mathcal{E}_2/\mathcal{E}_1 = N_2/N_1$;

$\mathcal{E}_2/(220\text{ V}) = 1/15$, which gives $\mathcal{E}_2 = \boxed{15\text{ V}}$.

(b) We find the current in the secondary from

$\mathcal{E}_2 = I_2R_2$;

$15\text{V} = I_2(15\ \Omega)$, which gives $I_2 = \boxed{1.0\text{ A}}$.

We find the current in the primary coil from

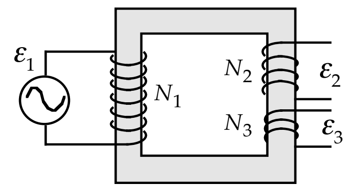
$N_2/N_1 = I_1/I_2$;

$1/15 = I_1/(1.0\text{ A})$, which gives $I_1 = \boxed{0.067\text{ A}}$.

(c) For the same power to be drawn directly from the 220-V generator, we have

$\mathcal{E}_1{}^2/R = I_2{}^2R_2 = I_2\mathcal{E}_2$;

$(220\text{ V})^2/R = (1.0\text{ A})(15\text{ V})$, which gives $R = \boxed{3.2\times10^3\ \Omega}$.



9. If we call the secondary windings N_2 and N_3, we have

$\mathcal{E}_2/\mathcal{E}_1 = N_2/N_1$ and $\mathcal{E}_3/\mathcal{E}_1 = N_3/N_1$, which combines to

$\mathcal{E}_3/\mathcal{E}_2 = N_3/N_2$;

$(11\text{ V})/(220\text{ V}) = N_3/(1000\text{ turns})$, which gives $N_3 = \boxed{50\text{ turns}}$.

10. For the two induced emfs, we can write

$\mathcal{E}_1 = -L\,dI_1/dt$, and $\mathcal{E}_2 = -M\,dI_1/dt$.

Because L and M are proportional to μ, $\boxed{\text{the ratio of the emf's is independent of } \mu}$.

11. We find the input frequency from

$X_C = 1/2\pi fC$;

$1.0\ \Omega = 1/[2\pi f(12\times10^{-6}\text{ F})]$, which gives $f = 1.3\times10^4\text{ Hz} = \boxed{13\text{ kHz}}$.

12. We find the current from

$I = V/R = (V_0/R)\sin(\omega t)$

$= [(130\text{ V})/(25\ \Omega)]\sin[2\pi(60\text{ Hz})t] = \boxed{5.2\sin(120\pi t)\text{ A}}$.

13. If we write the current as $I = I_0\sin(\omega t)$, the emf in the solenoid is

$\mathcal{E} = -L\,dI/dt = -LI_0\omega\cos(\omega t) = -\mathcal{E}_0\cos(\omega t)$.

We find the angular frequency from

$\mathcal{E}_0 = LI_0\omega = I_0X_L$;

$330\text{ V} = (2\text{ A})(15\times10^{-3}\text{ H})\omega$, which gives $\omega = \boxed{1.1\times10^4\text{ rad/s}}$.

14. If we write the current as $I = I_0 \sin(\omega t)$, the voltage across the capacitor is
$V_C = q/C = (1/C)\int I\,dt = -(I_0/C\omega)\cos(\omega t) = -V_0\cos(\omega t)$.
We find the maximum voltage from
$V_0 = I_0/C\omega = I_0 X_C$
$= (2.26\text{ A})/[(40 \times 10^{-6}\text{ F})2\pi(60\text{ Hz})] = \boxed{1.50 \times 10^2\text{ V}}$.

15. We find the capacitance from
$V_0 = I_0 X_C = I_0/C\omega$;
$95\text{ V} = (2.45\text{ A})/[C2\pi(180\text{ Hz})]$, which gives $C = 2.28 \times 10^{-5}\text{ F} = \boxed{22.8\ \mu\text{F}}$.

16. We find the inductive reactance from
$V_0 = I_0 X_L$;
$4\text{ V} = (0.065\text{ A})X_L$, which gives $X_L = 62\ \Omega$.
The inductance of the circuit is
$L = X_L/\omega = (62\ \Omega)/2\pi(220\text{ Hz}) = 45 \times 10^{-3}\text{ H} = \boxed{45\text{ mH}}$.

17. The maximum value of the capacitive reactance occurs at the minimum frequency:
$X_{C\max} = 1/\omega_{\min}C = 1/(300\text{ rad/s})(2 \times 10^{-6}\text{ F}) = \boxed{1.7 \times 10^3\ \Omega.}$
The maximum value of the inductive reactance occurs at the maximum frequency:
$X_{L\max} = \omega_{\max}L = (1000\text{ rad/s})(0.3\text{ H}) = \boxed{300\ \Omega}$.

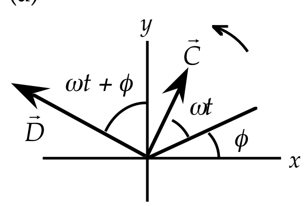
18. The current exists for positive and negative values of t. For simplicity, we find the positive values, recognizing that the sequences can be extended to negative values.
(a) For the current $I = I_0 \sin[\omega t + (\pi/4)]$, the peak current occurs when
$\sin[\omega t - (\pi/3)] = \pm 1$, or $\omega t - (\pi/3) = \pi/2, 3\pi/2, 5\pi/2, \ldots = (n - \tfrac{1}{2})\pi$, $n = 1, 2, 3, \ldots$.
The times are
$t = (1/\omega)[n - (1/6)]\pi = [1/2\pi(60\text{ Hz})][n - (1/6)]\pi(10^3\text{ ms/s})$, $n = 1, 2, 3, \ldots$, which gives
$t = \boxed{\ldots, 6.9\text{ ms}, 15.3\text{ ms}, 23.6\text{ ms}, \ldots}$.
(b) We find the peak voltage from
$V_0 = I_0 X_L = I_0\omega L$
$= (2.3\text{ A})[2\pi(60\text{ Hz})](0.25\text{ H}) = \boxed{217\text{ V}}$.
The voltage across the inductor is
$V = L\,dI/dt = L\omega I_0\cos(\omega t - \pi/3) = V_0\cos(\omega t - \pi/3)$, so the peak voltage occurs when
$\cos(\omega t - \pi/3) = \pm 1$, or $\omega t - \pi/3 = 0, \pi, 2\pi, 3\pi, \ldots = n\pi$, $n = 0, 1, 2, 3, \ldots$.
The times are
$t = (1/\omega)[n + (1/3)]\pi = [1/2\pi(60\text{ Hz})](n + 1/3)\pi(10^3\text{ ms/s})$, $n = 1, 2, 3, \ldots$, which gives
$t = \boxed{\ldots, 2.8\text{ ms}, 11.1\text{ ms}, 19.4\text{ ms}, \ldots}$.

19. Both the voltage and the current in the circuit are sinusoidal, so we have
$\langle V^2\rangle = V_0^2\langle\sin^2(\omega t)\rangle = \tfrac{1}{2}V_0^2$ and $\langle I^2\rangle = I_0^2\langle\cos^2(\omega t)\rangle = \tfrac{1}{2}I_0^2$.
We find the inductive reactance from
$V_0 = I_0 X_L$, or $\langle V^2\rangle = \langle I^2\rangle X_L^2$;
$(30\text{ V})^2 = (2\text{ A})^2 X_L^2$, which gives $X_L = \boxed{15\ \Omega}$.
We find the frequency from
$X_L = \omega L = 2\pi f L$;
$15\ \Omega = 2\pi f(25 \times 10^{-3}\text{ H})$, which gives $f = \boxed{95\text{ Hz}}$.

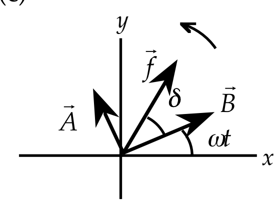
20. Because the different frequencies have different inductive reactance, we find the current from each term of the voltage and add them.
For an angular frequency of 400 rad/s, we have
$X_{L1} = \omega_1 L = (400 \text{ rad/s})(40 \times 10^{-6} \text{ H}) = 16 \times 10^{-3}\ \Omega$.
The maximum current is
$I_{01} = (0.3 \text{ V})/(0.016\ \Omega) = 19$ A, which lags the voltage by $\frac{1}{2}\pi$, so we have
$I_1 = (19 \text{ A}) \sin[(400 \text{ rad/s})t - \frac{1}{2}\pi]$.
For an angular frequency of 2700 rad/s, we have
$X_{L1} = \omega_1 L = (2700 \text{ rad/s})(40 \times 10^{-6} \text{ H}) = 0.11\ \Omega$.
The maximum current is
$I_{01} = (0.3 \text{ V})/(0.11\ \Omega) = 2.7$ A, which lags the voltage by $\frac{1}{2}\pi$, so we have
$I_1 = (2.7 \text{ A}) \sin[(2700 \text{ rad/s})t - \frac{1}{2}\pi]$.
The total current is
$I = I_1 + I_2 = \boxed{(19 \text{ A}) \sin[(400 \text{ s}^{-1})t - \frac{1}{2}\pi] + (2.7 \text{ A}) \sin[(2700 \text{ s}^{-1})t - \frac{1}{2}\pi]}$.

21. (a) We can write
$D \cos(\omega t + \phi) = D \sin[\omega t + \phi + (\pi/2)]$,
so $\boxed{\vec{D} \text{ is more advanced in phase}}$.
(b) The phase difference is $\pi/2$, or $\boxed{90°}$. $\boxed{\vec{D} \text{ leads}}$.
(c) We can write
$f(t) = A \cos(\omega t) + B \sin(\omega t)$
$= f \sin \delta \cos(\omega t) + f \cos \delta \sin(\omega t)$
$= f \sin(\omega t + \delta)$,
where $f = (A^2 + B^2)^{1/2}$, and $\boxed{\tan \delta = A/B}$.
Whether $\vec{f}$ leads or lags $\vec{C}$ depends on whether $\delta > \phi$, or $\delta < \phi$.

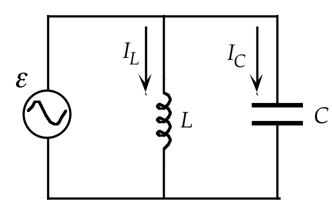
(a)



(c)



22. The voltage from the source is also the voltage across the inductor, so we have
$V_0 \sin(\omega t) = I_L\, dI_L/dt$.
The current is given by the integral of $\sin(\omega t)$:
$I_L \sim \int \sin(\omega t)\, dt \sim \boxed{-\cos(\omega t)}$.



23. For radio waves, we have
$c = f\lambda = \omega\lambda/2\pi$.
We find the capacitance from the natural frequency of the circuit:
$\omega_0 = (1/LC)^{1/2} = 2\pi c/\lambda_0$;
$[1/(14 \times 10^{-6} \text{ H})C]^{1/2} = 2\pi(3.0 \times 10^8 \text{ m/s})/(42 \text{ m})$, which gives $C = 3.5 \times 10^{-11} \text{ F} = \boxed{35 \text{ pF}}$.

24. We find the capacitance from the natural frequency of the circuit:
$\omega_0 = (1/LC)^{1/2} = 2\pi f$.
For the lowest frequency, we have
$[1/(12 \times 10^{-3} \text{ H})C_1]^{1/2} = 2\pi(540 \times 10^3 \text{ Hz})$, which gives $C_1 = 7.3 \times 10^{-12} \text{ F} = \boxed{7.3 \text{ pF}}$.
For the highest frequency, we have
$[1/(12 \times 10^{-3} \text{ H})C_2]^{1/2} = 2\pi(1600 \times 10^3 \text{ Hz})$, which gives $C_2 = 0.83 \times 10^{-12} \text{ F} = \boxed{0.83 \text{ pF}}$.
The capacitance range needed is 0.83 pF to 7.3 pF.

25. At the resonant frequency the impedance is the resistance:
$I_{\max} = V_0/Z = V_0/R$.
To double the current we have
$I_{\max 2}/I_{\max 1} = 2 = R_1/R_2$, which gives $\boxed{R_2 = \frac{1}{2}R_1}$.

26. For resonance, the driving frequency must be the natural frequency:

$\omega = \omega_0 = (1/LC)^{1/2}$;

$2\pi(750\text{ Hz}) = [1/(5 \times 10^{-3}\text{ H})C]^{1/2}$, which gives $C = 9.0 \times 10^{-6}\text{ F} = \boxed{9.0\ \mu\text{F}}$.

27. We find the maximum charge from

$Q_{\max} = I_{\max}/\omega = (100 \times 10^{-3}\text{ A})/2\pi(60\text{ Hz}) = \boxed{2.7 \times 10^{-4}\text{ C}}$.

The emf is

$V_0 = I_{\max}Z = (100 \times 10^{-3}\text{ A})(40\ \Omega) = \boxed{4.0\text{ V}}$.

28. Let the resonance frequency of the circuit be that of the Channel 6 audio signal:

$f_0 = 1/[2\pi(LC)^{1/2}]$; so

$L = 1/(4\pi^2 f_0^2 C) = 1/[4\pi^2 (87.75 \times 10^6\text{ Hz})^2(20 \times 10^{-12}\text{ F})] = \boxed{0.16\ \mu\text{H}}$.

$\boxed{29}$. The impedance of the circuit is $Z = [R^2 + (\omega L)^2]^{1/2}$, so the current in the circuit is $I = \mathcal{E}/Z$, and the voltage across the resistor is

$V_R = IR = \mathcal{E}R/Z = \mathcal{E}R/[R^2 + (\omega L)^2]^{1/2}$, which gives

$L = [(\mathcal{E}/V_R)^2 - 1]^{1/2}\,R/\omega = [(110\text{ V}/40\text{ V})^2 - 1]^{1/2}\,(150\ \Omega)/(60\text{ rad/s}) = \boxed{6.4\text{ H}}$.

30. (a) $X_C = 1/\omega C = 1/[2\pi(60\text{ Hz})(15.0 \times 10^{-6}\text{ F})] = \boxed{177\ \Omega}$.

$X_L = \omega L = 2\pi(60\text{ Hz})(30.0 \times 10^{-3}\text{ H}) = \boxed{11.3\ \Omega}$.

$Z = [(X_L - X_C)^2 + R^2]^{1/2} = \{[(11.3\ \Omega) - (177\ \Omega)]^2 + (18\ \Omega)^2\}^{1/2} = \boxed{167\ \Omega}$.

(b) $X_C = 1/\omega C = 1/2\pi(500\text{ Hz})(15.0 \times 10^{-6}\text{ F}) = \boxed{21.2\ \Omega}$.

$X_L = \omega L = 2\pi(500\text{ Hz})(30.0 \times 10^{-3}\text{ H}) = \boxed{94.2\ \Omega}$.

$Z = [(X_L - X_C)^2 + R^2]^{1/2} = [(94.2\ \Omega - 21.2\ \Omega)^2 + (18\ \Omega)^2]^{1/2} = \boxed{75.2\ \Omega}$.

(c) $X_C = 1/\omega C = 1/[2\pi(20.0 \times 10^3\text{ Hz})(15.00 \times 10^{-6}\text{ F})] = \boxed{0.531\ \Omega}$.

$X_L = \omega L = 2\pi(20.0 \times 10^3\text{ Hz})(30.0 \times 10^{-3}\text{ H}) = \boxed{3.77\text{ k}\Omega}$.

$Z = [(X_L - X_C)^2 + R^2]^{1/2} = [(3770\ \Omega - 0.531\ \Omega)^2 + (18\ \Omega)^2]^{1/2} = \boxed{3.77\text{ k}\Omega}$.

31. $X_C = 1/\omega C = 1/[2\pi(1200.0\text{ Hz})(2 \times 10^{-6}\text{ F})] = \boxed{66.3\ \Omega}$.

$X_L = \omega L = 2\pi(1200.0\text{Hz})(92 \times 10^{-3}\text{ H}) = \boxed{694\ \Omega}$.

$Z = [(X_L - X_C)^2 + R^2]^{1/2} = [(694\ \Omega - 66.3\ \Omega)^2 + (500\ \Omega)^2]^{1/2} = \boxed{802\ \Omega}$.

$Q_{\max} = V_0/\omega Z = (80\text{ V})/[2\pi(1200.0\text{Hz})(802\ \Omega)] = 1.3 \times 10^{-5}\text{ C} = \boxed{13\ \mu\text{C}}$.

$\tan\phi = (X_L - X_C)/R = (694\ \Omega - 66.3\ \Omega)/(500\ \Omega) = +1.26$, which gives $\phi = \boxed{+51.5°}$.

$I_{\max} = V_0/Z = (80\text{ V})/(802\ \Omega) = \boxed{0.10\text{ A}}$.

32. For a frequency of 1000 rad/s, we have the following values possible:

$X_{C1} = 1/\omega C_1 = 1/(1000\text{ rad/s})(1 \times 10^{-6}\text{ F}) = 1 \times 10^3\ \Omega$;

$X_{C2} = 1/\omega C_2 = 1/(1000\text{ rad/s})(100 \times 10^{-6}\text{ F}) = 10\ \Omega$;

$X_{L1} = \omega L_1 = (1000\text{ rad/s})(10 \times 10^{-3}\text{ H}) = 10\ \Omega$;

$X_{L2} = \omega L_2 = (1000\text{ rad/s})(25 \times 10^{-3}\text{ H}) = 25\ \Omega$;

$R_1 = 1 \times 10^3\ \Omega$; $R_2 = 3 \times 10^3\ \Omega$.

To minimize the impedance, $Z = [(X_L - X_C)^2 + R^2]^{1/2}$, we want to minimize $X_L - X_C$ and R so we choose C_2, L_1, and R_1, which gives

$Z_{\min} = \{[(10\ \Omega) - (10\ \Omega)]^2 + (1 \times 10^3\ \Omega)^2\}^{1/2} = \boxed{1 \times 10^3\ \Omega\text{, for } C_2 = 100\ \mu\text{F}, L_1 = 10\text{ mH}, R_1 = 1\text{ k}\Omega}$.

33. Because the emf is turned on at $t = 0$, we have

$$V = V_0 \sin(\omega t).$$

All elements have zero voltage at $t = 0$ s. 0.10000 seconds is 120 cycles, so any transient currents will have subsided, and the steady-state current will have been reached.

The voltage on the capacitor is

$$V_C = Q/C = -(Q_{max}/C)\cos(\omega t - \phi)$$
$$= -[(13 \times 10^{-6}\text{ C})/(2 \times 10^{-6}\text{ F})]\cos[2400\pi(0.10000\text{ s})(180°/\pi) - (+51.5°)] = \boxed{-4.05\text{ V}}.$$

The current in the circuit is

$$I = (V_0/Z)\sin(\omega t - \phi),$$

so the voltage across the inductor is

$$V_L = L\,\mathrm{d}I/\mathrm{d}t = +(LV_0/Z)\omega\cos(\omega t - \phi) = +(X_L V_0/Z)\cos(\omega t - \phi)$$
$$= [(694\ \Omega)(80\text{ V})/(802\ \Omega)]\cos[2400\pi(0.10000\text{ s})(180°/\pi) - (+51.5°)] = \boxed{+43.1\text{ V}}.$$

Note that the voltage across the resistor is

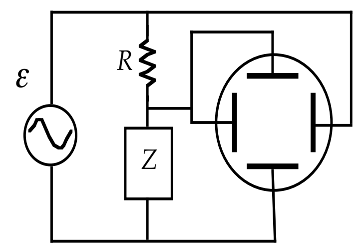
$$V_R = IR = (V_0 R/Z)\sin(\omega t - \phi)$$
$$= [(80\text{ V})(500\ \Omega)/(802\ \Omega)]\sin[2400\pi(0.10000\text{ s})(180°/\pi) - (+51.5°)]$$
$$= -39.0\text{ V},$$

so the sum of the voltages is 0, which is V at $t = 0.10000$ s.

34. We assume that the current is up, given by $I = I_0 \sin(\omega t)$.

For the connections shown, the horizontal voltage is

$$V_R = -RI = -RI_0\sin(\omega t) = -(12\ \Omega)I_0\sin(\omega t).$$



(a) The inductive reactance is

$$X_L = \omega L = 2\pi(60\text{ Hz})(0.95 \times 10^{-3}\text{ H}) = 0.36\ \Omega.$$

Because the voltage of the inductor leads the current, for the vertical voltage we have

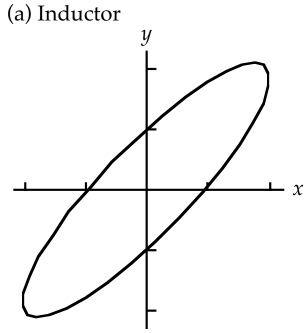
$$V_Z = V_r + V_L$$
$$= -rI_0\sin(\omega t) - X_L I_0\sin(\omega t + \tfrac{1}{2}\pi)$$
$$= -(0.650\ \Omega)I_0\sin(\omega t) - (0.36\ \Omega)I_0\sin(\omega t + \tfrac{1}{2}\pi).$$

The oscilloscope deflections are proportional to the voltages, or

$$x \propto -(12\ \Omega)\sin(\omega t);$$
$$y \propto -(0.650\ \Omega)\sin(\omega t) - (0.36\ \Omega)\sin(\omega t + \tfrac{1}{2}\pi),$$

With ωt as a parameter, this gives an ellipse.

(a) Inductor



(b) The capacitive reactance is

$$X_C = 1/\omega C = 1/2\pi(60\text{ Hz})(0.5 \times 10^{-3}\text{ F}) = \boxed{5.3\ \Omega}.$$

Because the voltage of the capacitor lags the current, for the vertical voltage we have

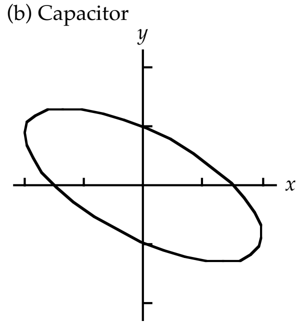
$$V_Z = V_r + V_C$$
$$= -rI_0\sin(\omega t) - X_C I_0\sin(\omega t - \tfrac{1}{2}\pi)$$
$$= -(0.650\ \Omega)I_0\sin(\omega t) - (5.3\ \Omega)I_0\sin(\omega t - \tfrac{1}{2}\pi).$$

The oscilloscope deflections are proportional to the voltages, or

$$x \propto -(12\ \Omega)\sin(\omega t);$$
$$y \propto -(0.650\ \Omega)\sin(\omega t) - (5.3\ \Omega)\sin(\omega t - \tfrac{1}{2}\pi),$$

With ωt as a parameter, this gives an ellipse.

(b) Capacitor



(c) If $X_L = 0$ or $X_C = 0$, the ellipse will degenerate to a straight line.
Thus a narrower ellipse corresponds to smaller reactance.
We could vary the angular frequency. If the ellipse becomes Narrower with increasing ω, the reactance must be capacitive.
If the ellipse becomes broader, the reactance must be inductive.

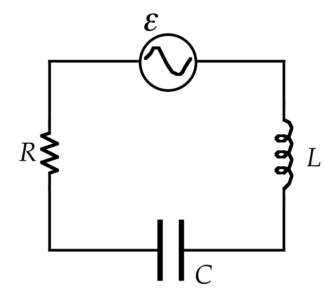
$\boxed{35}$. We find the reactances and the impedance:

$X_C = 1/\omega C = 1/2\pi(60\text{ Hz})(2\times10^{-6}\text{ F}) = 1.33\times10^3\ \Omega;$

$X_L = \omega L = 2\pi(60\text{ Hz})(0.8\text{ H}) = 302\ \Omega;$

$Z = [(X_L - X_C)^2 + R^2]^{1/2}$

$= [(302\ \Omega - 1.33\times10^3\ \Omega)^2 + (600\ \Omega)^2]^{1/2} = 1.19\times10^3\ \Omega.$



We find the maximum current from

$I_{\max} = V_0/Z = (110\text{ V})/(1.19\times10^3\ \Omega) = 0.092\text{ A} = \boxed{92\text{ mA}}.$

The maximum potential drops are

$V_{R\max} = I_{\max}R = (0.092\text{ A})(600\ \Omega) = \boxed{55.5\text{ V}}.$

$V_{C\max} = I_{\max}X_C = (0.092\text{ A})(1.33\times10^3\ \Omega) = \boxed{123\text{ V}}.$

$V_{L\max} = I_{\max}X_L = (0.092\text{ A})(302\ \Omega) = \boxed{27.9\text{ V}}.$

36. We find the resonant angular frequency from

$\omega_0 = (1/LC)^{1/2} = [1/(0.8\text{ H})(2\times10^{-6}\text{ F})]^{1/2} = \boxed{7.9\times10^2\text{ rad/s}}.$

At this frequency, $Z_0 = R$ and $I_{\max} = V_0/Z_0 = V_0/R$. To reduce the current to one-half, we have

$Z = 2Z_0$, or $[(X_L - X_C)^2 + R^2]^{1/2} = 2R$; $[(\omega L - 1/\omega C)^2 + R^2]^{1/2} = 2R.$

This is a quadratic equation for ω:

$L\omega^2 \pm R\omega\sqrt{3} - 1/C = (0.8\text{ H})\omega^2 \pm (600\ \Omega)(\sqrt{3})\omega - 1/(2\times10^{-6}\text{ F}) = 0$, which gives

$\omega = \boxed{3.7\times10^2\text{ rad/s} \quad\text{or}\quad 1.67\times10^3\text{ rad/s}}.$

37. With a scaling of $\frac{1}{2}$, the new value of the capacitance is

$C_2 = \varepsilon_0 A_2/d_2 = \varepsilon_0(\frac{1}{2})^2A_1/(\frac{1}{2}d_1) = \frac{1}{2}C_1.$

If the coils of the solenoid are tightly wrapped, reducing the wire size will increase the density of turns. With a scaling of $\frac{1}{2}$, the new value of the inductance is

$L_2 = \mu_0 n_2{}^2 A_2 \ell_2 = \mu_0(2n_1)^2(\frac{1}{2})^2A_1(\frac{1}{2}\ell_1) = \frac{1}{2}L_1.$

We find the new resonant frequency from

$\omega_2 = (1/L_2C_2)^{1/2} = [1/(\frac{1}{2}L)(\frac{1}{2}C_1)]^{1/2} = 2\omega_1$. The resonant frequency would $\boxed{\text{double}}$.

38. From the proposed solution, we find the other terms in Eq. 33–28:

$Q = -\,Q_{\max}\cos(\omega t - \phi);$

$\mathrm{d}Q/\mathrm{d}t = +\,Q_{\max}\omega\sin(\omega t - \phi);$

$\mathrm{d}^2Q/\mathrm{d}t^2 = +\,Q_{\max}\omega^2\cos(\omega t - \phi).$

When we substitute these in Eq. 34–28, we have

$V_0\sin(\omega t) - L\,\mathrm{d}^2Q/\mathrm{d}t^2 - R\,(\mathrm{d}Q/\mathrm{d}t) - Q/C = 0;$

$V_0\sin(\omega t) - LQ_{\max}\omega^2\cos(\omega t - \phi) - RQ_{\max}\omega\sin(\omega t - \phi) + (Q_{\max}/C)\cos(\omega t - \phi) = 0.$

We expand the sine and cosine functions to get

$V_0\sin(\omega t) - (LQ_{\max}\omega^2 + Q_{\max}/C)[\cos(\omega t)\cos\phi + \sin(\omega t)\sin\phi] -$

$RQ_{\max}\omega[\sin(\omega t)\cos\phi - \cos(\omega t)\sin\phi] = 0.$

When we combine all of the $\sin(\omega t)$ and $\cos(\omega t)$ terms, we get

$[V_0 + (-\,LQ_{\max}\omega^2 + Q_{\max}/C)\sin\phi - (RQ_{\max}\omega)\cos\phi]\sin(\omega t) +$

$[(-\,LQ_{\max}\omega^2 + Q_{\max}/C)\cos\phi + (RQ_{\max}\omega)\sin\phi]\cos(\omega t) = 0.$

For arbitrary times, this equation will be satisfied if each of the coefficients is zero:

$\cos(\omega t)$ term: $(-\,LQ_{\max}\omega^2 + Q_{\max}/C)\cos\phi + (RQ_{\max}\omega)\sin\phi = 0$, which gives

$\tan\phi = (L\omega - 1/\omega C)/R;$

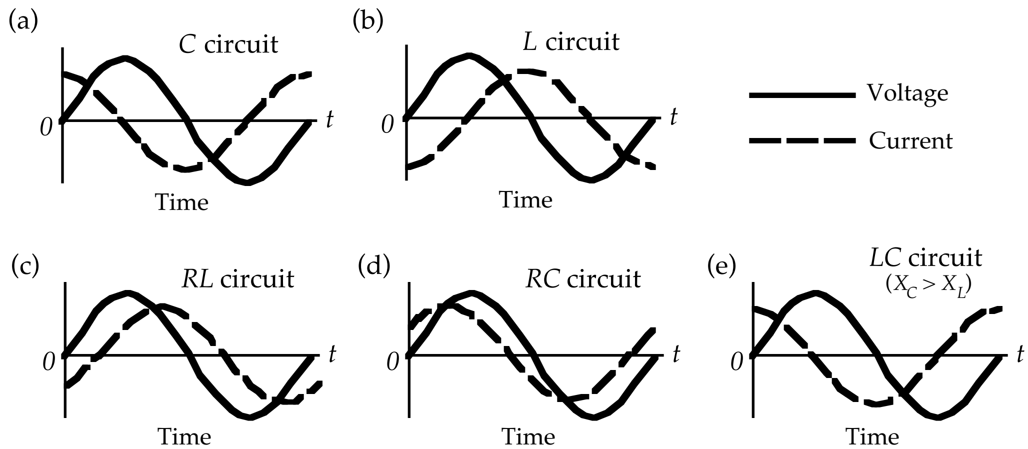
$\sin(\omega t)$ term: $V_0 + (-\,LQ_{\max}\omega^2 + Q_{\max}/C)\sin\phi - (RQ_{\max}\omega)\cos\phi = 0$, which gives

$Q_{\max} = V_0/[(L\omega^2 - 1/C)\sin\phi + R\omega\cos\phi],$

which can be reduced, by using the result for $\tan\phi$, to

$\boxed{Q_{\max} = V_0/\omega Z, \qquad \text{where } Z = [(L\omega)^2 - (1/\omega C)^2] + R^2]^{1/2}}.$

39.



40. For the purely resistive circuit, we have

$V_0 = I_{max}R;$

$80\text{V} = (3\text{ A})R$, which gives $R = 27\ \Omega$.

For the RC circuit, we have

$V_0 = I_{max}Z;$

$80\text{ V} = (1.5\text{ A})Z$, which gives $Z = 53\ \Omega$.

We find the capacitance from

$Z^2 = X_C^2 + R^2;$

$(53\ \Omega)^2 = X_C^2 + (27\Omega)^2$, which gives $X_C = 46\ \Omega$, or

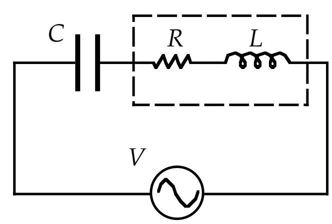
$C = 1/\omega X_C = 1/2\pi(60\text{ Hz})(46\ \Omega) = 58 \times 10^{-6}\text{ F} = \boxed{58\text{mF}}$.

We find the voltage drops from

$V_{C\max} = I_{\max}X_C = (1.5\text{ A})(46\ \Omega) = \boxed{69\text{ V}}$;

$V_{R\max} = I_{\max}R = (1.5\text{ A})(27\ \Omega) = \boxed{41\text{ V}}$.

41. We find the inductance from the resonant frequency:



$\omega_0^2 = 1/LC;$

$[2\pi(60\text{ Hz})]^2 = 1/[L(16 \times 10^{-6}\text{ F})]$, which gives $L = 0.44$ H.

At resonance the reactances are equal:

$X_C = X_L = \omega L = 2\pi(60\text{ Hz})(0.44\text{ H}) = 166\ \Omega$, and $Z = R = 30\ \Omega$.

The voltage across the capacitor is

$V_{C\max} = I_{\max}X_C = V_0X_C/Z = (12\text{ V})(166\ \Omega)/(30\ \Omega) = \boxed{66\text{ V}}$,

which is also the voltage across the inductance. Because the inductance voltage lags the resistance voltage, $V_{R\max} = I_{\max}R$, by $\pi/2$, we find the voltage across the inductor-resistor combination from

$V_{RL\max}^2 = V_{R\max}^2 + V_{L\max}^2 = (12\text{ V})^2 + (66\text{ V})^2$, which gives $V_{RL\max} = \boxed{67\text{ V}}$.

42. For the maximum voltages to be equal, we have

$I_{\max}R = I_{\max}X_C = I_{\max}X_L$, or $R = X_C = X_L$, which gives

$1/\omega C = R$, or $C = \boxed{1/2\pi fR}$;

$\omega L = R$, or $L = \boxed{R/2\pi f}$.

43. We find the inductance from the resonant frequency:

$\omega_0^2 = 1/LC;$

$[2\pi(180 \times 10^3\text{ Hz})]^2 = 1/L(70 \times 10^{-9}\text{ F})$, which gives $L = 1.1 \times 10^{-5}\text{ H} = \boxed{11\ \mu\text{H}}$.

44. We find the equivalent values for each of the components:

two resistors in series: $R_{eq} = R_1 + R_2$;

two capacitors in series: $1/C_{eq} = 1/C_1 + 1/C_2$;

two inductors in series: $L_{eq} = L_1 + L_2$.

For the reactances, we have

$X_{Ceq} = 1/\omega C_{eq} = 1/\omega C_1 + 1/\omega C_2 = X_{C1} + X_{C2}$;

$X_{Leq} = \omega L_{eq} = \omega L_1 + \omega L_2 = X_{L1} + X_{L2}$.

The total impedance is

$Z_{total} = [(X_{Leq} - X_{Ceq})^2 + R_{eq}^2]^{1/2} = \{[(X_{L1} + X_{L2}) - (X_{C1} + X_{C2})]^2 + (R_1 + R_2)^2\}^{1/2}$.

45. With $V(t) = V_0 \sin(\omega t)$ and $I(t) = I_0 \sin(\omega t - \phi)$, we find the phase difference between the two from

$\tan \phi = (X_L - X_C)/R$.

(a) For a circuit with a resistor and capacitor, we have

$\tan \phi = (X_L - X_C)/R = -X_C/R$.

Thus $\phi < 0$, and the current leads the voltage.

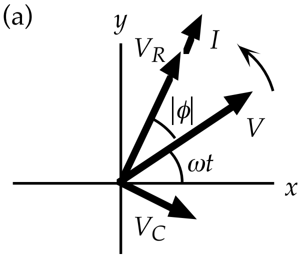
The potential drop across the resistor is

$V_R = RI = RI_0 \sin(\omega t - \phi)$.

The potential drop across the capacitor is

$V_C = Q/C = -(Q_0/C) \cos(\omega t - \phi)$.

(a)



(b) For a circuit with a resistor and inductor, we have

$\tan \phi = (X_L - X_C)/R = +X_L/R$.

Thus $\phi > 0$, and the current lags the voltage.

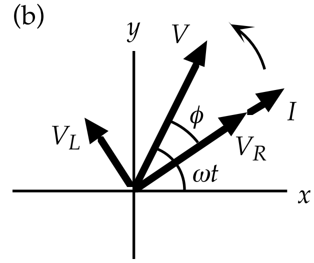
The potential drop across the resistor is

$V_R = RI = RI_0 \sin(\omega t - \phi)$.

The potential drop across the inductor is

$V_L = L\, dI/dt = LI_0 \cos(\omega t - \phi)$.

(b)



46. For the power dissipated in a resistor circuit, we have

$\langle P \rangle = \langle I^2 \rangle R = \tfrac{1}{2} I_{max}^2 R = \tfrac{1}{2}(1.36\text{ A})^2(120\ \Omega) = \boxed{111\text{ W}}$.

47. We find the maximum current from

$\langle P \rangle = \langle I^2 \rangle R = \tfrac{1}{2} I_{max}^2 R$;

$150\text{ W} = \tfrac{1}{2} I_{max}^2 (12\ \Omega)$, which gives $I_{max} = 5.0$ A.

Because this is the current at $t = 0$, we have

$I = I_0 \cos(\omega t)$

$= (5.0\text{ A}) \cos[2\pi(75\text{ Hz})(0.04\text{ s})] = (5.0\text{ A}) \cos(6\pi) = \boxed{5.0\text{ A}}$.

Note that 0.04 s is 3 cycles.

48. From the charge on the capacitor,

$Q = CV = CV_0 \sin(\omega t)$, we have

$I = dQ/dt = CV_0\omega \cos(\omega t)$.

The instantaneous power is

$P = IV$

$= [CV_0\omega \cos(\omega t)][V_0 \sin(\omega t)] = \boxed{\omega C V_0^2 \sin(\omega t) \cos(\omega t)} = \tfrac{1}{2}\omega C V_0^2 \sin(2\omega t)$.

The average power is

$\langle P \rangle = \tfrac{1}{2}\omega C V_0^2 \langle \sin(2\omega t) \rangle = \boxed{0}$, because the average value of a sine function over a period is zero.

We know this is so because there is no loss of energy in the capacitor; the charge oscillates.

49. The average power in an RLC circuit is
$\langle P\rangle = \frac{1}{2}V_0{}^2R/Z = \frac{1}{2}V_0{}^2R/[(X_L - X_C)^2 + R^2]^{1/2} = \frac{1}{2}V_0{}^2R/\{[(\omega L) - (1/\omega C)]^2 + R^2\}^{1/2}$.
(a) When $\omega \to \infty$, $Z \to \omega L$, and we have
$\boxed{\langle P\rangle \to \frac{1}{2}V_0{}^2R/\omega L \to 0}$.
The current is a very small because there is a large induced emf in the inductor.
(b) When $\omega \to 0$, $Z \to 1/\omega C$, and we have
$\boxed{\langle P\rangle \to \frac{1}{2}V_0{}^2R\omega C \to 0}$.
The current is a very small because the system approaches a DC circuit, which would have $\boxed{\text{no current through the capacitor}}$.

50. (a) We find the resistance from
$\langle P\rangle = \langle V^2\rangle/R = \frac{1}{2}V_{\text{max}}{}^2/R$;
$8.0 \times 10^2\text{ W} = \frac{1}{2}(110\text{ V})^2/R$, which gives $R = \boxed{7.6\ \Omega}$.
(b) We find the rms current from
$\langle P\rangle = I_{\text{rms}}{}^2R$;
$8.0 \times 10^3\text{ W} = I_{\text{rms}}{}^2(7.6\ \Omega)$, which gives $I_{\text{rms}} = \boxed{10.3\text{ A}}$.
(c) The maximum current is
$I_{\text{max}} = I_{\text{rms}}\sqrt{2} = (10.3\text{ A})\sqrt{2} = \boxed{14.5\text{ A}}$.

51. (a) For a pure capacitive circuit, $R = 0$, $X_L = 0$, $Z = X_C$, so we have
$\cos\phi = R/Z = 0/X_C = \boxed{0}$.
(b) For a pure inductive circuit, $R = 0$, $X_C = 0$, $Z = X_L$, so we have
$\cos\phi = R/Z = 0/X_L = \boxed{0}$.
(c) For a pure resistive circuit, $X_C = 0$, $X_L = 0$, $Z = R$, so we have
$\cos\phi = R/Z = R/R = \boxed{1}$.

52. From the current
$I = I_0 \sin(\omega t - \phi)$, we get
$V_L = L\,\mathrm{d}I/\mathrm{d}t = -LI_0\omega\cos(\omega t - \phi)$.
The instantaneous power is
$P = IV_L$
$= [I_0\sin(\omega t - \phi)][-LI_0\omega\cos(\omega t - \phi)] = -\omega LI_0{}^2\sin(\omega t - \phi)\cos(\omega t - \phi) = -\frac{1}{2}\omega CV_0{}^2\sin[2(\omega t - \phi)]$.
The average power is
$\langle P\rangle = -\frac{1}{2}\omega CV_0{}^2\langle\sin[2(\omega t - \phi)]\rangle = 0$, because the average value of a sine function over 2 cycles is zero.

53. (a) For RL circuits,
$X_C = 0$, $Z = (X_L{}^2 + R^2)^{1/2}$, so we have
$\cos\phi = R/Z = \boxed{R/(X_L{}^2 + R^2)^{1/2}}$.
(b) For RC circuits,
$X_L = 0$, $Z = (X_C{}^2 + R^2)^{1/2}$, so we have
$\cos\phi = R/Z = \boxed{R/(X_C{}^2 + R^2)^{1/2}}$.
(c) For LC circuits,
$R = 0$, $Z = X_L - X_C$, so we have
$\cos\phi = R/Z = 0/(X_L - X_C) = \boxed{0}$.

54. The impedance is

$Z = \mathcal{E}_{max} / I_{max} = 170\ \text{V}/0.4\ \text{A} = 425\ \Omega = \boxed{0.4\ \text{k}\Omega}$.

From $P = I_{rms}^2 R = (I_{max} /\sqrt{2})^2 R$ we get

$R = 2P/I_{max}^2 = 2(18\ \text{W})/(0.4\ \text{A})^2 = 225\ \Omega = \boxed{0.2\ \text{k}\Omega}$.

Finally, from $Z = [R^2 + (\omega L)^2]^{1/2}$ we solve for L:

$L = (Z^2 - R^2)^{1/2}/\omega = [(425\ \Omega)^2 - (225\ \Omega)^2]^{1/2}/[2\pi(60\ \text{Hz})] = 0.956\ \text{H} \approx \boxed{1\ \text{H}}$.

55. (a) The resonance frequency is

$f_0 = \omega_0/2\pi = 1/[2\pi(LC)^{1/2}] = (1/2\pi)[(170 \times 10^{-6}\ \text{F})(24 \times 10^{-3}\ \text{H})]^{-1/2} = \boxed{79\ \text{Hz}}$.

(b) From $P = I^2R = (\mathcal{E}/Z)^2R$ we have

$P(\omega)/P(\omega_0) = [Z(\omega_0)/Z(\omega)]^2 = (R\omega_0)^2/[(R\omega)^2 + L^2(\omega^2 - \omega_0^2)^2]$.

For $\omega = 1.01\omega_0$, $P(\omega) = 0.02\ P(\omega_0)$; so

$(R\omega_0)^2/[(1.01R\omega_0)^2 + L^2(1.01^2 - 1)^2\ \ \omega_0^4] = 0.02$;

$R = \boxed{0.034\ \Omega}$.

56. We find the voltage amplitude from

$V_0 = V_{rms}\sqrt{2} = (78\ \text{V})\sqrt{2} = \boxed{110\ \text{V}}$.

For the current, we have

$I_{rms} = V_{rms}/Z = (78\ \text{V})/(20\ \Omega) = \boxed{3.9\text{A}}$;

$I_0 = V_0/Z = (110\ \text{V})/(20\ \Omega) = \boxed{5.5\ \text{A}}$.

57. We find the impedance of the coil from

$\langle P\rangle = (V_{rms1}^2/Z_1)\cos\phi$;

$200\ \text{W} = [(110\ \text{V})^2/Z_1](0.6)$, which gives $Z_1 = 36.3\ \Omega$.

The resistance of the coil is $R = Z_1 \cos\phi = (36.3\ \Omega)(0.6) = 21.8\ \Omega$.

We find the inductive reactance from

$Z_1^2 = X_L^2 + R^2$;

$(36.3\ \Omega)^2 = X_L^2 + (21.8\ \Omega)^2$, which gives $X_L = 29.0\ \Omega$.

When the capacitor is added to the circuit, we find the impedance from

$\langle P\rangle = V_{rms2}^2 R/Z_2^2$;

$200\ \text{W} = (220\ \text{V})^2(21.8\ \Omega)/Z_2^2$, which gives $Z_2 = 72.6\ \Omega$.

We find the capacitive reactance from

$Z_2^2 = (X_L - X_C)^2 + R^2$;

$(72.6\ \Omega)^2 = [(29.0\ \Omega) - X_C]^2 + (21.8\ \Omega)^2$, which gives $X_C = 98.2\ \Omega$.

The capacitance is $C = 1/\omega X_C = 1/2\pi(60\ \text{Hz})(98.2\ \Omega) = 2.7 \times 10^{-5}\ \text{F} = \boxed{27\ \mu\text{F}}$.

To maintain the same power factor with the same resistance by adding a capacitor, we have

$R/Z_1 = R/Z_3$, or $Z_1 = Z_3$;

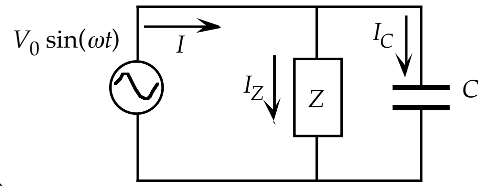
$(X_L - X_{C3})^2 = X_L^2$, which has two solutions:

$X_{C3} = 0$, which is the original circuit, and $X_{C3} = 2X_L = 2(29.0\ \Omega) = 58.0\ \Omega$.

The capacitance is

$C_3 = 1/\omega X_{C3} = 1/2\pi(60\ \text{Hz})(58.0\ \Omega) = 4.6 \times 10^{-5}\ \text{F} = \boxed{46\ \mu\text{F}}$.

58. We will take all currents and voltages as rms values.



(a) Because the capacitor is placed in parallel, its current leads the voltage by $\frac{1}{2}\pi$; we find the total current as the sum of the currents in the branches:

$I_Z = I_0 \sin(\omega t - \phi) = I_0 \sin(\omega t) \cos \phi - I_0 \cos(\omega t) \sin \phi$;

$I_C = I_1 \sin(\omega t + \frac{1}{2}\pi) = I_1 \cos(\omega t)$;

$I = I_Z + I_C = I_0 \sin(\omega t) \cos \phi - I_0 \cos(\omega t) \sin \phi + I_1 \cos(\omega t)$

$= I_0 \cos \phi \sin(\omega t) + (I_1 - I_0 \sin \phi) \cos(\omega t)$.

To get a power factor of 1.0, we want the current to be in phase with the voltage, $V = V_0 \sin(\omega t)$, which means we want the $\cos(\omega t)$ term to be zero:

$I_1 = I_0 \sin \phi = (120\text{ A}) \sin 40° = 77\text{ A}$.

We find the required capacitance from

$X_C = 1/\omega C = V_0/I_1$;

$1/2\pi(60\text{ Hz})C = (220\text{ V})/(77\text{ A})$, which gives $C = 9.3 \times 10^{-4}\text{ F} = \boxed{930\ \mu\text{F}}$.

(b) The rms current becomes

$I = I_0 \cos \phi = (120\text{ A}) \cos 40° = \boxed{92\text{ A}}$.

(c) We find the total power from

$P = IV_0 = (92\text{ A})(220\text{ V}) = 2.02 \times 10^4\text{ W} = \boxed{20.2\text{ kW}}$.

$\boxed{59}$. We find the impedance and resistance of the motor from

$\langle P \rangle = (V_0^2/2Z_1) \cos \phi$;

$5 \times 10^3\text{ W} = [(220\text{ V})^2/2Z_1](0.80)$, which gives $Z_1 = 3.87\ \Omega$;

$R_1 = Z_1 \cos \phi = (3.87\ \Omega)(0.80) = 3.1\ \Omega$.

The current in the circuit is

$I_0 = V_0/Z_1 = (220\text{ V})/(3.87\ \Omega) = 56.8\text{ A}$.

If we include the transmission line, we find the impedance from

$Z_2^2 = (X_L - X_C)^2 + (R_1 + R_2)^2 = Z_1^2 - R_1^2 + (R_1 + R_2)^2$;

$= (3.87\ \Omega)^2 - (3.1\ \Omega)^2 + (3.10\ \Omega + 2.5\ \Omega)^2$, which gives $Z_2 = 6.07\ \Omega$.

The voltage that must be supplied at the input is

$V_{02} = I_0 Z_2 = (56.8\text{ A})(6.07\ \Omega) = \boxed{345\text{ V}}$.

The power that must be supplied at the input is

$\langle P_2 \rangle = V_{02}^2(R_1 + R_2)/2Z_2^2 = (345\text{ V})^2(3.1\ \Omega + 2.5\ \Omega)/[2(6.07\ \Omega)^2] = 9.1 \times 10^3\text{ W} = \boxed{9.1\text{ kW}}$.

60. We find the resistance of the device from

$\langle P \rangle = V_0^2 R/2Z^2 = (V_0^2/2R)(\cos \phi)^2$;

$4 \times 10^3\text{ W} = [(220\text{ V})^2/2R](0.85)^2$, which gives $R = 4.4\ \Omega$.

We find the rms current from

$\langle P \rangle = I_{\text{rms}}^2 R$;

$4 \times 10^3\text{ W} = I_{\text{rms}}^2(4.4\ \Omega)$, which gives $I_{\text{rms}} = 30.2\text{ A}$.

The power lost in the transmission line is

$P_{\text{lost}} = I_{\text{rms}}^2 R_t = (30.2\text{ A})^2(15\ \Omega) = 1.4 \times 10^4\text{ W} = \boxed{14\text{ kW}}$.

Because this is almost the power delivered, better transmission cables should be used.

$\boxed{61}$. We find the impedance from

$V_0 = I_0 Z$; $220\text{ V} = (80\text{ A})Z$, which gives $Z = 2.75\ \Omega$.

For a power factor of 0.55, the maximum power is

$P_{\max 1} = I_0^2 Z \cos \phi_1 = (80\text{ A})^2(2.75\ \Omega)(0.55) = 9.7 \times 10^3\text{ W} = \boxed{9.7\text{ kW}}$.

For a power factor of 0.95, the maximum power is

$P_{\max 2} = I_0^2 Z \cos \phi_2 = (80\text{ A})^2(2.75\ \Omega)(0.95) = 16.7 \times 10^3\text{ W} = \boxed{16.7\text{ kW}}$.

62. The capacitive reactance is

$X_C = 1/\omega C = 1/[2\pi(60\text{ Hz})(4.5 \times 10^{-6}\text{ F})] = \boxed{590\ \Omega}$.

We find the impedance from

$Z^2 = X_C^2 + R^2$

$= (590\ \Omega)^2 + (20\ \Omega)^2$, which gives

$Z = \boxed{590\ \Omega}$.

We find the current from

$V_0 = I_0 Z$;

$110\text{ V} = I_0(590\ \Omega)$, which gives

$I_0 = \boxed{0.186\text{ A}}$.

We find the power from

$\langle P\rangle = I_{\text{rms}}^2 R = \frac{1}{2} I_0^2 R$;

$= \frac{1}{2}(0.186\text{ A})^2(20\ \Omega) = \boxed{6.9\text{W}}$.

The power factor is

$\cos\phi = R/Z = (20\ \Omega)/(590\ \Omega) = \boxed{0.034}$.

The added inductance has a reactance of

$X_L = \omega L = 2\pi(60\text{ Hz})(0.035\text{ H}) = 13.2\ \Omega$.

We find the new impedance from

$Z^2 = (X_L - X_C)^2 + R^2$

$= (13.2\ \Omega - 590\ \Omega)^2 + (20\ \Omega)^2$, which gives

$Z = \boxed{577\ \Omega}$.

We find the current from

$V_0 = I_0 Z$;

$110\text{ V} = I_0(577\ \Omega)$, which gives $\quad I_0 = \boxed{0.191\text{ A}}$.

We find the power from

$\langle P\rangle = I_{\text{rms}}^2 R = \frac{1}{2} I_0^2 R$;

$= \frac{1}{2}(0.191\text{ A})^2(20\ \Omega) = \boxed{0.365\text{ W}}$.

The power factor is

$\cos\phi = R/Z = (20\ \Omega)/(577\ \Omega) = \boxed{0.035}$.

63. Before the capacitor is short-circuited

$P = I_{\text{rms}}^2 R$; so

$R = P/I_{\text{rms}}^2 = (200\text{ W})/(3.00\text{ A})^2 = \boxed{22.2\ \Omega}$. Also

$I_{\text{rms}} = \mathcal{E}_{\text{rms}}/Z = 220\text{ V}/[R^2 + (X_L - X_C)^2]^{1/2} = 3.00\text{ A}$.

After the capacitor is shorted out

$I_{\text{rms}} = \mathcal{E}_{\text{rms}}/Z = 220\text{ V}/(R^2 + X_L^2)^{1/2} = 2.20\text{ A}$. $R = 22.2\ \Omega$; $X_L = 98\ \Omega$; $X_C = 168\ \Omega$.

The last two equations yields $X_L = \boxed{98\ \Omega}$ and $X_C = \boxed{168\ \Omega}$. (Note that $X_C > X_L$).

64. The resonance frequency is

$f_0 = 1/[2\pi(LC)^{1/2}]$; so

$L = (1/2\pi f_0)^2/C$

$= 1/[2\pi(2.2 \times 10^6\text{ Hz})]^2/(6.6 \times 10^{-12}\text{ F}) = \boxed{0.79\text{ mH}}$.

According to Eq. 32-43

$\Delta\omega = R/L = 2\pi[2(3.0 \times 10^3\text{ Hz})]$; so

$R = L\,\Delta\omega = (0.79\text{ mH})\,4\pi(3.0 \times 10^3\text{ Hz}) = \boxed{30\ \Omega}$.

65. (a) We find the reactances and the impedance:

$X_C = 1/\omega C = 1/[2\pi(60\text{ Hz})(20 \times 10^{-6}\text{ F})] = 133\ \Omega$;

$X_L = \omega L = 2\pi(60\text{ Hz})(10 \times 10^{-3}\text{ H}) = 3.77\ \Omega$;

$Z = [(X_L - X_C)^2 + R^2]^{1/2} = [(3.77\ \Omega - 133\ \Omega)^2 + (50\ \Omega)^2]^{1/2} = 139\ \Omega$.

The power factor is

$\cos\phi = R/Z = (50\ \Omega)/(139\ \Omega) = 0.36$.

We find the power absorbed from

$\langle P\rangle = I_{\text{rms}}{}^2 Z\ \cos\phi = (V_{\text{rms}}{}^2/Z)\cos\phi$

$= [(110\text{ V})^2/(139\ \Omega)](0.36) = \boxed{31\text{ W}}$.

(b) If the resistance is halved, we find the new impedance:

$Z = [(X_L - X_C)^2 + R^2]^{1/2} = [(3.77\ \Omega - 133\ \Omega)^2 + (25\ \Omega)^2]^{1/2} = 132\ \Omega$.

The power factor is

$\cos\phi = R/Z = (25\ \Omega)/(132\ \Omega) = 0.19$.

We find the power absorbed from

$\langle P\rangle = I_{\text{rms}}{}^2 Z\ \cos\phi = (V_{\text{rms}}{}^2/Z)\cos\phi$

$= [(110\text{ V})^2/(132\ \Omega)](0.19) = \boxed{17\text{ W}}$.

(c) The maximum power drawn is

$P_{\max} = 2\langle P\rangle = 2(17\text{ W}) = \boxed{34\text{ W}}$;

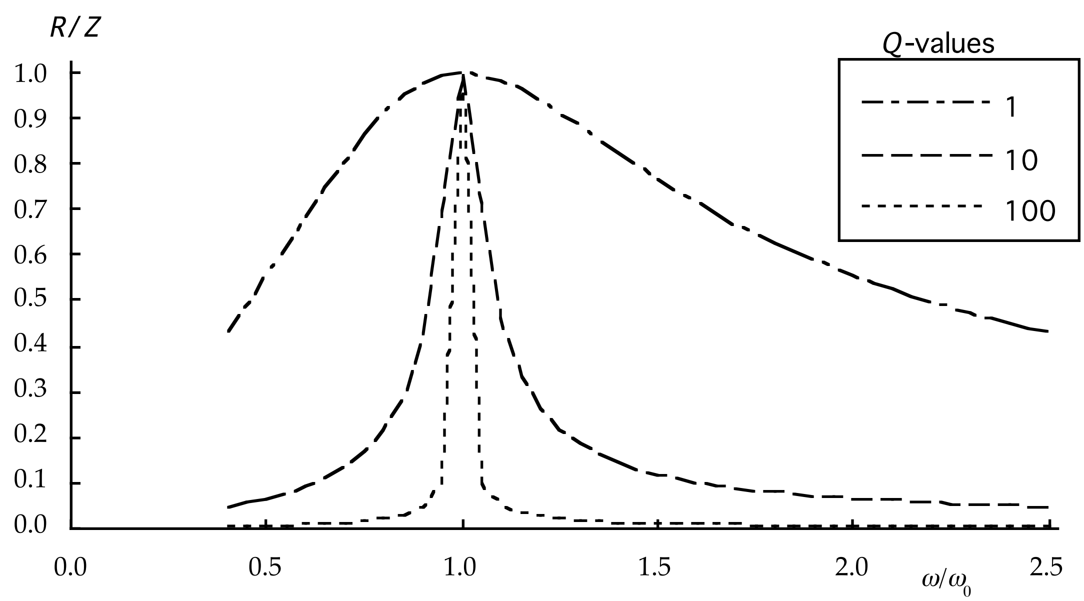
66. We use the expression for the resonance frequency in the definition of impedance:

$$\frac{R}{Z} = \frac{R}{\sqrt{(X_L - X_C)^2 + R^2}} = \frac{R}{\sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2}} = \frac{R}{\sqrt{\left(\omega L - \frac{L}{\omega CL}\right)^2 + R^2}}$$

$$= \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{\omega_0^2 L}{\omega}\right)^2}} = \frac{1}{\sqrt{1 + \left(\frac{\omega_0 L}{R}\right)^2\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2}}$$

$$= \frac{1}{\sqrt{1 + Q^2\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2}},\ \text{where } Q = \frac{\omega_0 L}{R}.$$

67.



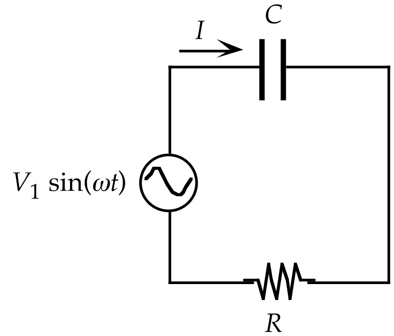
68. For small values of the resistance, the bandwidth is

$\Delta\omega = R/L$, so we have

$\Delta\omega/\omega = R/L\omega = (R/L\omega_0)(\omega_0/\omega) = (1/Q)(\omega_0/\omega)$.

When the resistance is small, $\omega \simeq \omega_0$, so we have $\Delta\omega/\omega \simeq 1/Q$.

69. For the RC circuit, we have



$X_C = 1/\omega C$,

$Z = (X_C{}^2 + R^2)^{1/2}$,

$I = V_1/Z$, and $V_C = IX_C = V_1X_C/Z$.

(a) $X_C = 1/[2\pi(100 \text{ Hz})(5 \times 10^{-9} \text{ F})] = 3.2 \times 10^5\ \Omega$,

$Z = [(3.2 \times 10^5\ \Omega)^2 + (120\ \Omega)^2]^{1/2} = 3.2 \times 10^5\ \Omega$,

$V_C = (0.20 \text{ V})(3.2 \times 10^5\ \Omega)/(3.2 \times 10^5\ \Omega) = \boxed{0.20 \text{ V}}$.

(b) $X_C = 1/[2\pi(10^5 \text{ Hz})(5 \times 10^{-9} \text{ F})] = 3.2 \times 10^2\ \Omega$,

$Z = [(3.2 \times 10^2\ \Omega)^2 + (120\ \Omega)^2]^{1/2} = 3.4 \times 10^2\ \Omega$,

$V_C = (0.20 \text{ V})(3.2 \times 10^2\ \Omega)/(3.4 \times 10^2\ \Omega) = \boxed{0.19 \text{ V}}$.

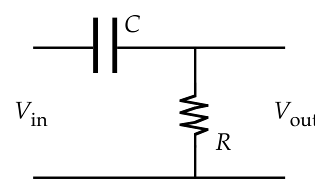
(c) $X_C = 1/2\pi(10 \times 10^6 \text{ Hz})(5 \times 10^{-9} \text{ F}) = 3.2\ \Omega$,

$Z = [(3.2\ \Omega)^2 + (120\ \Omega)^2]^{1/2} = 120\ \Omega$,

$V_C = (0.20 \text{ V})(3.2 \times 10^{-3}\ \Omega)/(120\ \Omega) = \boxed{5.3 \times 10^{-6} \text{ V}}$.

This is a low-pass filter (filters high frequencies).

70. For a high-pass filter, we put the capacitor and resistor in series and use the voltage across the resistor as the output. At low frequency, the reactance of the capacitor will be large and the voltage across the resistor will be small. At high frequencies, the reactance of the capacitor will be small and the voltage across the resistor will be almost the input voltage. If we set $X_C = R$, we have



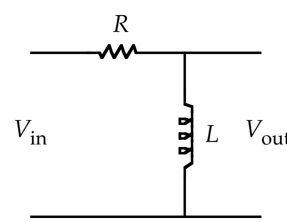
$Z = (X_C{}^2 + R^2)^{1/2} = R\sqrt{2}$, so $V_{\text{out}} = IR = V_{\text{in}}R/Z = V_{\text{in}}\sqrt{2}$,

which we will use as the condition at our cut-off frequency:

$R = 1/2\pi fC$, which gives $RC = 1/2\pi f = 1/2\pi(8 \times 10^3 \text{ Hz}) = 2.0 \times 10^{-5} \text{ s}^{-1}$.

We can satisfy this requirement with many combinations, e. g., $R = 20\ \Omega$ and $C = 1\ \mu$F.

71. For a high-pass filter, we put the inductor and resistor in series and use the voltage across the inductor as the output. At low frequency, the reactance of the inductor will be small and the voltage across the inductor will be small. At high frequencies, the reactance of the inductor will be large and the voltage across the resistor will be small. If we set $X_L = R$, we have



$Z = (X_L{}^2 + R^2)^{1/2} = X_L\sqrt{2}$, so $V_{\text{out}} = IX_L = V_{\text{in}}X_L/Z = V_{\text{in}}\sqrt{2}$,

which we will use as the condition at our cut-off frequency:

$R = 2\pi fL$, which gives $R/L = 2\pi f = 2\pi(8 \times 10^3 \text{ Hz}) = \boxed{5.0 \times 10^4 \text{ s}^{-1}}$.

We can satisfy this requirement with many combinations, e. g., $R = 2.5$ kΩ and $L = 50$ mH.

72. When the reactance terms are equal and opposite, we have

$\langle P\rangle = \mathcal{E}_{\text{rms}}{}^2R_2/(R_1 + R_2)^2$.

We want to maximize the average power, so we set $d\langle P\rangle/dR_2 = 0$:

$$\frac{d\langle P\rangle}{dR_2} = \mathcal{E}_{\text{rms}}^2\left[\frac{1}{(R_1 + R_2)^2} + \frac{(-2)R_2}{(R_1 + R_2)^3}\right] = \mathcal{E}_{\text{rms}}^2\frac{R_1 + R_2 - 2R_2}{(R_1 + R_2)^3} = 0,$$

which gives $R_1 = R_2$.

73. The current through the diode for one cycle is

$I = I_0 \sin(\omega t),\ 0 < t < T/2;$

$I = 0,\ T/2 < t < T$, where $T = 2\pi/\omega$.

The average current is

$$\langle I \rangle = \frac{\int_0^{T/2} I_0 \sin(\omega t)\, dt}{\int_0^T dt} = \frac{\int_0^{\pi} I_0 \sin(\omega t)\, d(\omega t)}{\int_0^{2\pi} d(\omega t)} = \frac{I_0(-\cos\theta)\Big|_0^{\pi}}{2\pi} = \frac{1}{\pi} I_0.$$

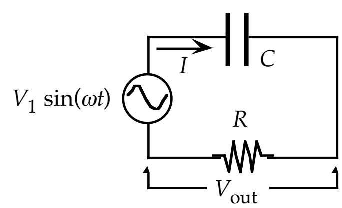
We find the rms current from

$$\langle I^2 \rangle = \frac{\int_0^{T/2} I_0^2 \sin^2(\omega t)\, dt}{\int_0^T dt} = \frac{\int_0^{\pi} I_0^2 \sin^2(\omega t)\, d(\omega t)}{\int_0^{2\pi} d(\omega t)} = \frac{I_0^2(\pi/2)}{2\pi} = \frac{1}{4} I_0^2, \text{ which gives}$$

$I_{rms} = \boxed{\tfrac{1}{2} I_0}.$

74. The impedance of the circuit is

$Z = (X_C^2 + R^2)^{1/2} = [(1/\omega C)^2 + R^2)]^{1/2}.$



The input voltage is $V_{in} = I_0 Z$, and the output voltage is

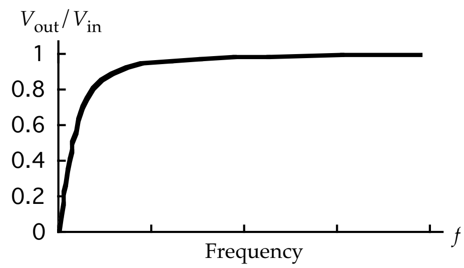
$V_{out} = I_0 R$, which gives

$V_{out}/V_{in} = R/Z = R/[(1/\omega C)^2 + R^2)]^{1/2}.$

At low frequency, $X_C \to \infty$, $Z \to \infty$, $I_0 \to 0$, so $V_{out} \to 0$.

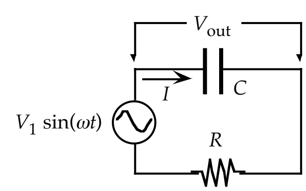
At high frequency, $X_C \to 0$, $Z \to R$, $I_0 \to V_{in}/R$, so $V_{out} \to V_{in}$.

Thus at low frequencies, X_C is large, so the current and $V_{out} = IR$ are small.



75. The impedance of the circuit is

$Z = (X_C^2 + R^2)^{1/2} = [(1/\omega C)^2 + R^2]^{1/2}.$



The input voltage is $V_{in} = I_0 Z$, and the output voltage is

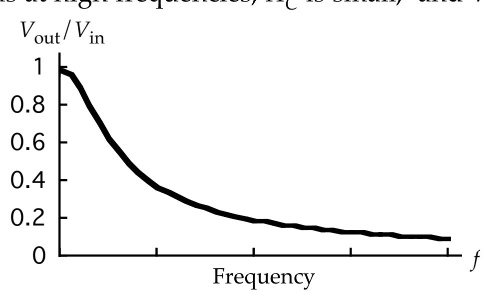
$V_{out} = I_0 X_C$, which gives

$V_{out}/V_{in} = X_C/Z = (1/\omega C)/[(1/\omega C)^2 + R^2)]^{1/2}.$

At low frequency, $X_C \to \infty$, $\boxed{Z \to X_C}$, so $V_{out} \to V_{in}$.

At high frequency, $X_C \to 0$, $\boxed{Z \to R}$, so $V_{out} \to 0$.

Thus at high frequencies, X_C is small, and V_{out} is small.



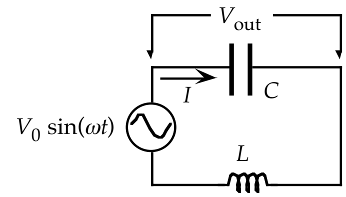
76. (a) With $X_L >> X_C$, the impedance of the circuit is

$$Z = [(X_L - X_C)^2]^{1/2} \simeq X_L.$$

The current is $I_0 = V_0/Z \simeq V_0/X_L$.

The output voltage is

$$V_{\text{out}} = V_C = I_0 X_C \simeq (X_C/X_L)V_0.$$



(b) We can treat the emf as a superposition of AC and DC components. From part (a), we see that

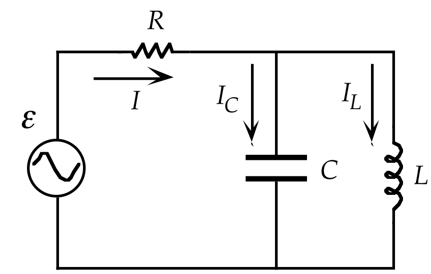
$V_{\text{out}} = (X_C/X_L)V_0 << 1$, so there is very little AC output.

For the DC component, the voltage across the inductor is

$V_L = L\,dI/dt = 0$, so $V_{\text{out}} = V_C = V_{\text{in}}$.

77. For the junction equation, we have

$$I = I_C + I_L.$$



For the loop equations, we have

left loop: $V_0 \cos(\omega t) - IR - Q/C = 0$;

right loop: $V_0 \cos(\omega t) - IR - L\,(dI_L/dt) = 0$.

Using our trial solution, $I = I_0 \cos(\omega t + \phi)$, we find the capacitor current from

$$Q = C[V_0 \cos(\omega t) - I_0 R \cos(\omega t + \phi)];$$

$$I_C = dQ/dt = -\omega C V_0 \sin(\omega t) + \omega C I_0 R \sin(\omega t + \phi) = -(V_0/X_C)\sin(\omega t) + (I_0 R/X_C)\sin(\omega t + \phi).$$

We find the inductor current from

$$dI_L/dt = (V_0/L)\cos(\omega t) - (I_0 R/L)\cos(\omega t + \phi);$$

$$I_L = \int (dI_L/dt)\,dt = (V_0/\omega L)\sin(\omega t) - (I_0 R/\omega L)\sin(\omega t + \phi) + \text{constant}$$

$$= (V_0/X_L)\sin(\omega t) - (I_0 R/X_L)\sin(\omega t + \phi) + \text{constant}.$$

By using the appropriate expansion of the trigonometric functions, we have

$$I = I_0[\cos(\omega t)\cos\phi - \sin(\omega t)\sin\phi],$$

$$I_C = -(V_0/X_C)\sin(\omega t) + (I_0 R/X_C)[\sin(\omega t)\cos\phi + \cos(\omega t)\sin\phi],$$

$$I_L = +(V_0/X_L)\sin(\omega t) - (I_0 R/X_L)[\sin(\omega t)\cos\phi + \cos(\omega t)\sin\phi] + \text{constant}.$$

For the junction equation to be satisfied for arbitrary t, the coefficients of $\sin(\omega t)$ and $\cos(\omega t)$ must satisfy the junction equation, which gives

$$\text{constant} = 0,$$

$$-I_0 \sin\phi = V_0(1/X_L - 1/X_C) + I_0 R(1/X_C - 1/X_L)\cos\phi,$$

$$+I_0 \cos\phi = I_0 R(1/X_C - 1/X_L)\sin\phi.$$

When we combine these equations, we get

$$\tan\phi = X_C X_L/(X_L - X_C)R, \text{ which gives}$$

$$X_{\text{eq}} = X_C X_L/(X_L - X_C).$$

This is the equivalent reactance of the two parallel elements. Because I_C leads the V_C and I_L lags V_L, there is a π phase difference between the two, which gives the negative sign in the denominator. The impedance of the circuit is

$$Z = (X_{\text{eq}}^2 + R^2)^{1/2}.$$

When we use these results in the current equations, we obtain $I_0 = V_0/Z$, so the three currents are

$$I = \boxed{(V_0/Z)\cos(\omega t + \phi)},$$

$$I_C = \boxed{(V_0/X_C)[(R/Z)\sin(\omega t + \phi) - \sin(\omega t)]},$$

$$I_L = \boxed{(V_0/X_L)[-(R/Z)\sin(\omega t + \phi) + \sin(\omega t)]}.$$

To see if there is a resonance frequency, we look at the impedance:

$$Z = \{[X_C X_L/(X_L - X_C)]^2 + R^2\}.$$

If $X_L = X_C$, $Z \to \infty$, so $I_0 \to 0$, and $I \to 0$. This means $I_C = -I_L$. There could be a large current in the LC circuit when $\omega^2 = 1/LC$, so $\boxed{\text{there is a resonant frequency}}$.

78. We assume that the two circuits are hooked up to the same emf. Since the inductor and capacitor do not consume any net power, for the circuit at the top the average power consumed is simply

$\langle P_1\rangle = \mathcal{E}_{\text{rms}}{}^2/R_1$.

For the circuit at the bottom, we first find the rms current I_{rms} that flows through the resistor R_2:

$I_{\text{rms}} = \mathcal{E}_{\text{rms}}/[(\omega L)^2 + R_2{}^2]^{1/2}$, so the average power consumed on R_2 is

$\langle P_2\rangle = I_{\text{rms}}{}^2 R_2 = \mathcal{E}_{\text{rms}}{}^2 R_2/[(\omega L)^2 + R_2{}^2]$.

Equate the expressions for $\langle P_1\rangle$ and $\langle P_2\rangle$ to obtain

$\mathcal{E}_{\text{rms}}{}^2/R_1 = \mathcal{E}_{\text{rms}}{}^2 R_2/[(\omega L)^2 + R_2{}^2]$, which gives

$R_1 R_2 = R_2^2 + (\omega L)^2 = R_2^2 + L/C$;

$\boxed{R_1 = R_2 + L/R_2C}$.

79. (a) For the capacitors, we have

$V_{C1} = I_0X_{C1} = I_0/\omega C_1 = (1.55\text{ A})/2\pi(8000\text{ Hz})(6\times10^{-6}\text{ F}) = \boxed{5.14\text{ V}}$;

$V_{C2} = I_0X_{C2} = I_0/\omega C_2 = (1.55\text{ A})/2\pi(8000\text{ Hz})(14\times10^{-6}\text{ F}) = \boxed{2.20\text{ V}}$.

(b) For the inductor, we have

$V_L = I_0X_L = (1.55\text{ A})(1.51\ \Omega) = \boxed{2.34\text{ V}}$.

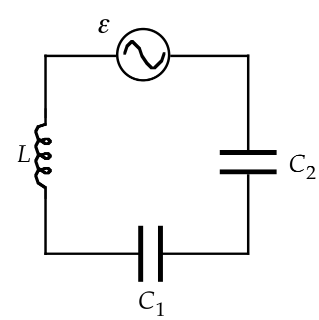
Note that, because there is a π phase difference between the capacitors and the inductor, we have

$V_{C1} + V_{C2} - V_L = V_0$.

80. The equivalent capacitance of the two capacitors in series is

$C = C_1C_2/(C_1 + C_2)$

$= (4\ \mu\text{F})(9\ \mu\text{F})/(4\ \mu\text{F} + 9\mu\text{F}) = 2.8\ \mu\text{F}$.



The reactances are

$X_C = 1/\omega C = 1/[2\pi(600\text{ Hz})(2.8\times10^{-6}\text{ F})] = 95\Omega$;

$X_L = \omega L = 2\pi(600\text{ Hz})(70\times10^{-6}\text{ H}) = 0.26\ \Omega$.

The impedance of the circuit is

$Z = |X_L - X_C| = |(95\ \Omega) - (0.26\ \Omega)| = 95\ \Omega$.

(a) We find the maximum current from

$V_0 = I_0Z$;

$4\text{V} = I_0(95\ \Omega)$, which gives $I_0 = \boxed{0.042\text{A}}$.

(b) We have

$\omega_0 = (1/LC)^{1/2} = [1/(70\times10^{-6}\text{ H})(2.8\times10^{-6}\text{ F})]^{1/2} = 7.1\times10^4\text{ rad/s}$,

so the resonant frequency is

$f_0 = \omega_0/2\pi = (7.1\times10^4\text{ rad/s})/2\pi = \boxed{1.1\times10^4\text{ Hz}}$.

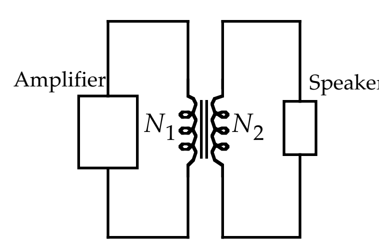
81. Starting from Eq. 33-31, we have

$\tan\phi = (X_L - X_C)/R = (\omega L - 1/\omega C)/R = (\omega - 1/\omega LC)L/R$

$= (\omega - \omega_0^2/\omega)L/R = (\omega^2 - \omega_0^2)L/\omega R$.

Here we used $X_L = \omega L$, $X_C = 1/\omega C$, and $\omega_0^2 = 1/LC$.

82. If we let 1 represent the amplifier and 2 represent the speaker, for the transformer we have



$V_2/V_1 = N_2/N_1$, and $I_2/I_1 = N_1/N_2$.

The current in each component is determined by V and Z:

$V_1 = I_1Z_1$ and $V_2 = I_2Z_2$.

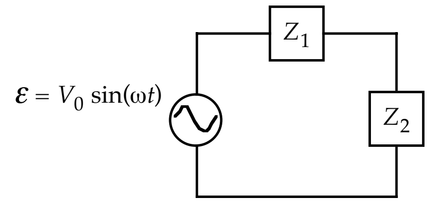
If we form the ratio, we have

$V_2/V_1 = (I_2/I_1)(Z_2/Z_1)$; $N_2/N_1 = (N_1/N_2)(Z_2/Z_1)$;

$(N_1/N_2)^2 = Z_1/Z_2 = (3{,}000\ \Omega)/(8\ \Omega)$, which gives $N_1/N_2 = \boxed{19}$.

83. The reactance of the capacitor is

$X_C = 1/\omega C = 1/2\pi(3000\text{ Hz})(2 \times 10^{-6}\text{ F}) = 26.5\ \Omega.$



The total impedance of the circuit is

$Z = [X_C^{\,2} + (R_1 + R_2)^2]^{1/2}$

$= [(26.5\ \Omega)^2 + (15\ \Omega + 8\ \Omega)^2]^{1/2} = 35.1\ \Omega.$

The current in the circuit is

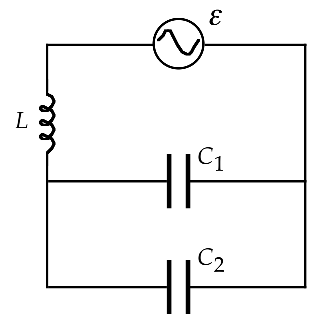
$I_0 = V_0/Z = (3\text{ V})/(35.3\ \Omega) = 8.5 \times 10^{-2}\text{ A},$ and $I_{\text{rms}} = I_0/\sqrt{2}.$

The power dissipated in Z_2 is

$P_2 = I_{\text{rms}}^{\,2}R_2 = \tfrac{1}{2}(8.5 \times 10^{-2}\text{ A})^2(8\ \Omega) = 2.9 \times 10^{-2}\text{ W} = \boxed{29\text{ mW}}.$

84. The equivalent capacitance of the two capacitors in parallel is

$C = C_1 + C_2 = 20\ \mu\text{F} + 30\ \mu\text{F} = 50\ \mu\text{F}.$



The reactances are

$X_{C1} = 1/\omega C_1 = 1/[2\pi(400\text{ Hz})(20 \times 10^{-6}\text{ F})] = 20\ \Omega;$

$X_{C2} = 1/\omega C_2 = 1/[2\pi(400\text{ Hz})(30 \times 10^{-6}\text{ F})] = 13\ \Omega;$

$X_C = 1/\omega C = 1/[2\pi(400\text{ Hz})(50 \times 10^{-6}\text{ F})] = 8.0\ \Omega;$

$X_L = \omega L = 2\pi(400\text{ Hz})(10 \times 10^{-3}\text{ H}) = 25\ \Omega.$

The impedance of the circuit is

$Z = [(X_L - X_C)^2]^{1/2} = |25\ \Omega - 8.0\ \Omega| = 17\ \Omega.$

(a) The maximum current in the circuit, which is the maximum current in the inductor, is

$I_0 = I_L = V_0/Z = (12\text{ V})/(17\ \Omega) = \boxed{0.71\text{ A}}.$

The voltage across the equivalent capacitance is the voltage across each capacitor:

$I_0X_C = I_{C1}X_{C1} = I_{C2}X_{C2};$

$(0.71\text{ A})(8.0\ \Omega) = I_{C1}(20\ \Omega) = I_{C2}(13\ \Omega),$ which gives $I_{C1} = \boxed{0.28\text{ A}},$ and $I_{C2} = \boxed{0.44\text{ A}}.$

(b) The resonant frequency is

$f_0 = \omega_0/2\pi = (1/2\pi)/(LC)^{1/2} = (1/2\pi)/[(10 \times 10^{-3}\text{ H})(50 \times 10^{-6}\text{ F})]^{1/2} = \boxed{225\text{ Hz}}.$

85. (a) For each capacitor, the maximum voltage is

$V_C = V_{C1} = V_{C2} = I_0X_C = (0.71\text{ A})(8.0\ \Omega) = \boxed{5.7\text{ V}}.$

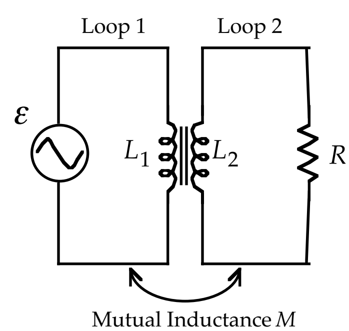
(b) The maximum voltage across the inductor is

$V_L = I_0X_L = (0.71\text{ A})(25\ \Omega) = \boxed{18\text{ V}}.$

86. We write a loop equation for each loop:

loop 1: $\mathcal{E} - L_1(\mathrm{d}I_1/\mathrm{d}t) - M(\mathrm{d}I_2/\mathrm{d}t) = 0;$

loop 2: $-M(\mathrm{d}I_1/\mathrm{d}t) - L_2(\mathrm{d}I_2/\mathrm{d}t) - I_2R = 0.$



87. We find the inductance from the resonant frequency:

$\omega_0 = 2\pi f_0 = (1/LC)^{1/2};$

$2\pi(18 \times 10^6\text{ Hz}) = \{1/[L(33 \times 10^{-12}\text{ F})]\}^{1/2},$ which gives $L = 2.4 \times 10^{-6}\text{ H} = \boxed{2.4\ \mu\text{H}}.$

We find the resistance from the width of the power vs. frequency curve:

$\Delta\omega = 2\pi\,\Delta f = R/L;$

$2\pi(40 \times 10^3\text{ Hz}) = R/(2.4 \times 10^{-6}\text{ H}),$ which gives $R = \boxed{0.060\ \Omega}.$

88. Because the resistor and capacitor are in parallel, the maximum voltage is the same across each:

$V_0 = I_R R = I_C X_C$;

110 V $= I_R(62\ \Omega) = I_C(3\Omega)$, which gives $I_R = 1.77$ A and $I_C = 36.7$ A.

These maximum currents do not occur at the same time; there is a 90° phase difference because the capacitor current leads the voltage. The maximum current of the combination is

$I_0 = (I_R^2 + I_C^2)^{1/2} = [(1.77\text{ A})^2 + (36.7\text{ A})^2]^{1/2} = \boxed{37\text{ A}}$.

89. The reactance of the capacitor is

$X_C = 1/\omega C = 1/[2\pi(60\text{ Hz})(15 \times 10^{-6}\text{ F})] = 177\ \Omega$.

The impedance of the circuit is

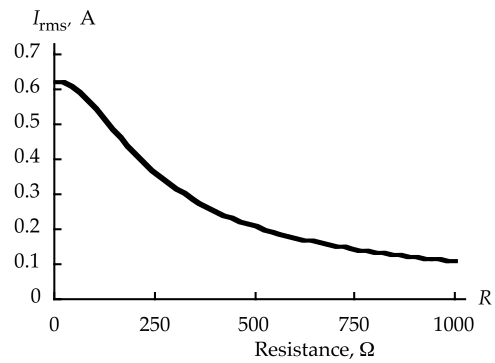
$Z = (X_C^2 + R^2)^{1/2} = [(177\ \Omega)^2 + R^2]^{1/2}$,

so the rms current is

$I_{\text{rms}} = V_{\text{rms}}/Z = (110\text{ V})/[(177\ \Omega)^2 + R^2]^{1/2}$.

The power delivered to the circuit is the power dissipated in the resistor:

$P = I_{\text{rms}}^2 R = V_{\text{rms}}^2 R/Z^2 = V_{\text{rms}}^2 R/(X_C^2 + R^2)$.



To find the value of R for which the power is maximum, we set $dP/dR = 0$:

$$dP/dR = [V_{\text{rms}}^2/(X_C^2 + R^2)] + [V_{\text{rms}}^2 R(-2R)/(X_C^2 + R^2)^2]$$
$$= V_{\text{rms}}^2(X_C^2 + R^2 - 2R^2)/(X_C^2 + R^2)^2 = V_{\text{rms}}^2(X_C^2 - R^2)/(X_C^2 + R^2)^2 = 0,$$

which gives $R = X_C = \boxed{177\ \Omega}$.

90. (a) The capacitive reactance is

$X_C = 1/\omega C = 1/[2\pi(60\text{ Hz})(35 \times 10^{-6}\text{ F})] = 75.7\ \Omega$.

Because the elements are in series, the maximum current in the circuit is the maximum current in the capacitor:

$V_{C\max} = I_0 X_C$;

1200 V $= I_0(75.7\ \Omega)$, which gives $I_0 = 15.8$ A.

(b) We find the minimum impedance of the circuit from

$I_{0\max} = V_0/Z_{\min} = (V_{\text{rms}}\sqrt{2})/Z_{\min}$;

15.8 A $= [(220\text{ V})\sqrt{2}]/Z_{\min}$, which gives $Z_{\min} = 19.6\ \Omega$.

For the minimum impedance, we have

$Z_{\min}^2 = (X_L - X_C)_{\min}^2 + R^2$;

$(19.6\ \Omega)^2 = (X_L - X_C)_{\min}^2 + (10\ \Omega)^2$, which gives

$|(X_L - X_C)|_{\min} = 16.9\ \Omega$.

For a fixed value of X_C, there are two regions where $Z > Z_{\min}$.

If $X_L < X_C$, we have

$X_C - X_{L\max} = |(X_L - X_C)|_{\min}$, or $X_{L\max} = 75.7\ \Omega - 16.9\ \Omega = 58.7\ \Omega$.

For this condition, the maximum value of the self-inductance is

$L_{\max} = X_{L\max}/\omega = (58.7\ \Omega)/2\pi(60\text{ Hz}) = 0.155\text{ H} = 155\text{ mH}$.

If $X_L > X_C$, we have

$X_{L\min} - X_C = |(X_L - X_C)|_{\min}$, or $X_{L\min} = 75.7\ \Omega + 16.9\ \Omega = 92.6\ \Omega$.

For this condition, the minimum value of the self-inductance is

$L_{\min} = X_{L\min}/\omega = (92.6\ \Omega)/[2\pi(60\text{ Hz})] = 0.245\text{ H} = 245\text{ mH}$.

The two ranges at which the inductance can be safely set are

30 mH $< L <$ 155 mH, and 245 mH $< L <$ 300 mH.

Note that the two ranges are separated by the resonance peak, so the inductance cannot be changed from one range to the other with the power on.

91. (a) For the complex charge, we have

$Q_c(t) = CV_c(t) = CV_0 e^{i\omega t}$.

For the complex current, we have

$I_c(t) = dQ_c(t)/dt = i\omega CV_0 e^{i\omega t}$.

(b) For the current, we have

$I = \mathrm{Im}[I_c(t)] = \mathrm{Im}\{i\omega CV_0\ [\cos(\omega t) + i\sin(\omega t)]\} = \mathrm{Im}[i\omega CV_0 \cos(\omega t) + i^2\omega CV_0 \sin(\omega t)]$

$= \mathrm{Im}[i\omega CV_0 \cos(\omega t) - \omega CV_0 \sin(\omega t)] = \omega CV_0 \cos(\omega t)$.

92. (a) (i) We find the complex current by direct integration of the loop equation:

$I_c(t) = \int dI_c(t) = (1/L)\int V_c(t)\,dt = (V_0/L)\int e^{i\omega t}\,dt$

$= (V_0/i\omega L)e^{i\omega t} = \boxed{-i(V_0/\omega L)e^{i\omega t}}$.

(ii) We assume the complex current is $I_c(t) = I_0 e^{i\omega t}$ and differentiate:

$V_c(t) = L\,dI_c(t)/dt$;

$V_0 e^{i\omega t} = LI_0(i\omega)e^{i\omega t}$, which gives $I_0 = V_0/i\omega L = -i(V_0/\omega L)$, which is the result from (i).

(b) For the current, we have

$I = \mathrm{Im}[I_c(t)] = \mathrm{Im}[-i(V_0/\omega L)\cos(\omega t) - i^2(V_0/\omega L)\sin(\omega t)]$

$= \mathrm{Im}[-i(V_0/\omega L)\cos(\omega t) + (V_0/\omega L)\sin(\omega t)] = -(V_0/\omega L)\cos(\omega t) = (V_0/\omega L)\sin(\omega t - \tfrac{1}{2}\pi)$.

93. (a) We differentiate the defined $Q_c(t) = -iQ_{0c}e^{i\omega t}$:

$I_c(t) = dQ_c(t)/dt = -i^2\omega Q_{0c}\,e^{i\omega t} = \omega Q_{0c}\,e^{i\omega t}$;

$dI_c(t)/dt = i\omega^2 Q_{0c}\,e^{i\omega t}$.

When we make the substitutions in the circuit equation, we get

$$V_c(t) = L\frac{dI_c(t)}{dt} + \frac{Q_c(t)}{C} + RI_c(t);$$

$$V_{0c}e^{i\omega t} = iL\omega^2 Q_{0c}e^{i\omega t} - \frac{iQ_{0c}}{C}e^{i\omega t} + R\omega Q_{0c}e^{i\omega t}.$$

Every term contains $e^{i\omega t}$, so we get

$V_{0c} = iL\omega^2 Q_{0c} - iQ_{0c}/C + R\omega\,Q_{0c}$, or $Q_{0c} = (V_{0c}/\omega)[i(L\omega - 1/\omega C) + R]^{-1}$.

(b) For the complex charge and complex current, we have

$$Q_c(t) = -iQ_{0c}e^{i\omega t} = \frac{-iV_{0c}e^{i\omega t}}{\omega\left[i\left(L\omega - \frac{1}{\omega C}\right) + R\right]};$$

$$I_c(t) = \omega\frac{V_{0c}e^{i\omega t}}{\omega\left[i\left(L\omega - \frac{1}{\omega C}\right) + R\right]} = \frac{V_{0c}e^{i\omega t}}{\left[i\left(L\omega - \frac{1}{\omega C}\right) + R\right]}.$$

We simplify the factor in brackets with the definitions $X_L = L\omega$, and $X_C = 1/\omega C$.

The magnitude of the complex number in the brackets is

$Z = [(X_L - X_C)^2 + R^2]^{1/2}$.

If we divide numerator and denominator by Z, we make the following definitions

$\cos\phi = R/Z$, and $\sin\phi = (X_L - X_C)/Z$;

which allows us to express the denominator as a complex exponential:

$[R + i(X_L - X_C)]/Z = \cos\phi + i\sin\phi = e^{i\phi}$.

We can write the complex charge and current as

$Q_c(t) = -i(V_0/\omega Z)e^{i\omega t}/e^{i\phi} = -i(V_0/\omega Z)e^{i(\omega t - \phi)}$;

$I_c(t) = (V_0/Z)e^{i\omega t}/e^{i\phi} = (V_0/Z)e^{i(\omega t - \phi)}$.

(c) For the charge we have

$Q = \mathrm{Im}[Q_c(t)] = \mathrm{Im}[-i(V_0/\omega Z)\cos(\omega t - \phi) - i^2(V_0/\omega Z)\sin(\omega t - \phi)]$

$= \mathrm{Im}[-i(V_0/\omega Z)\cos(\omega t - \phi) + (V_0/\omega Z)\sin(\omega t - \phi)] = \boxed{-(V_0/\omega Z)\cos(\omega t - \phi)}$.

For the current, we have

$I = \mathrm{Im}[I_c(t)] = \mathrm{Im}[(V_0/Z)\cos(\omega t - \phi) + i(V_0/Z)\sin(\omega t - \phi)] = \boxed{(V_0/Z)\ \sin(\omega t - \phi)}$.

94. We define the complex impedance and use the result for the complex current:

$$Z_c(t) = V_c(t)/I_c(t) = V_0e^{i\omega t}/(V_0/Z)e^{i(\omega t-\phi)} = Ze^{i\phi},$$

which is independent of time.

The magnitude of the complex impedance is

$$|Z_c(t)| = Z|e^{i\phi}| = Z(\cos^2\phi + \sin^2\phi)^{1/2} = Z = [(X_L - X_C)^2 + R^2]^{1/2} = [(L\omega - 1/\omega C)^2 + R^2]^{1/2}.$$

95. We differentiate the proposed solution:

$$Q_c(t) = Q_{0c}e^{i\omega t}$$

$$I_c(t) = \frac{dQ_c(t)}{dt} = i\omega Q_{0c}e^{i\omega t};$$

$$\frac{dI_c(t)}{dt} = i^2\omega^2Q_{0c}e^{i\omega t} = -\omega^2Q_{0c}e^{i\omega t}.$$

When we make the substitutions in the circuit equation, we get

$$L\frac{dI_c(t)}{dt} + \frac{Q_c(t)}{C} + RI_c(t) = 0;$$

$$-L\omega^2Q_{0c}e^{i\omega t} + \frac{Q_{0c}}{C}e^{i\omega t} + iR\omega Q_{0c}e^{i\omega t}.$$

Every term contains $Q_{0c}e^{i\omega t}$, so we get

$$-L\omega^2 + 1/C + i\omega R = 0, \quad \text{or} \quad \omega^2 - i(R/L)\omega - 1/LC = 0.$$

The solution of this quadratic equation for ω is

$$\omega = \boxed{[(1/LC) - (R^2/4L^2)]^{1/2} + i(R/2L)}.$$

We make the following definitions:

$\alpha = R/2L$, and $\omega' = [(1/LC) - (R^2/4L^2)]^{1/2}$, so we can write

$$\omega = \omega' + i\alpha.$$

The complex charge becomes

$$Q_c(t) = Q_{0c}\,e^{i\omega t} = Q_{0c}\,e^{-\alpha t}e^{i\omega' t},$$

which has the exponential decay and the sinusoidal oscillation at angular frequency ω'.